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OF THE
ROYAL METEOROLOGICAL SOCIETY

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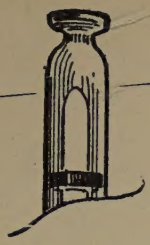

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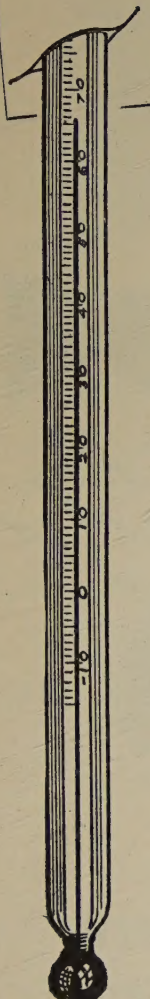

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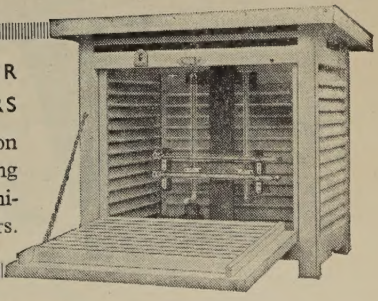

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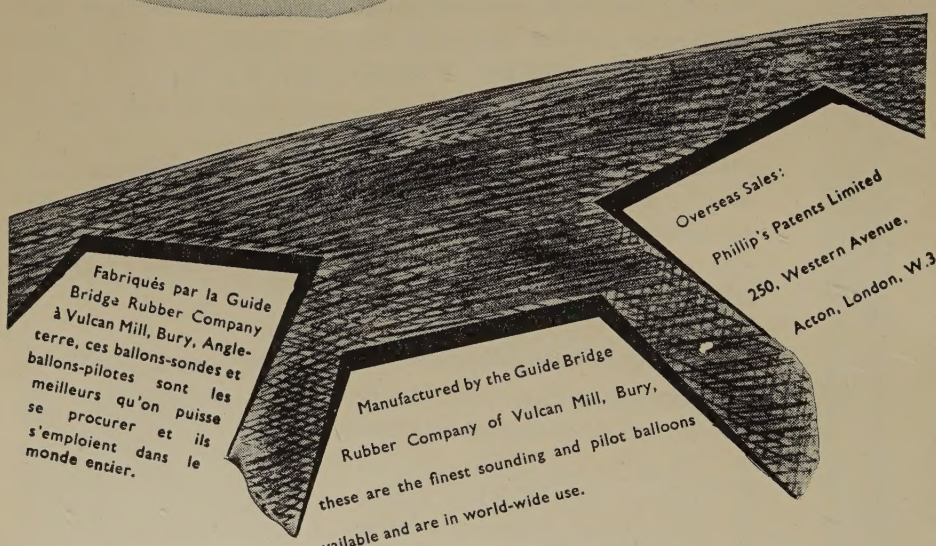

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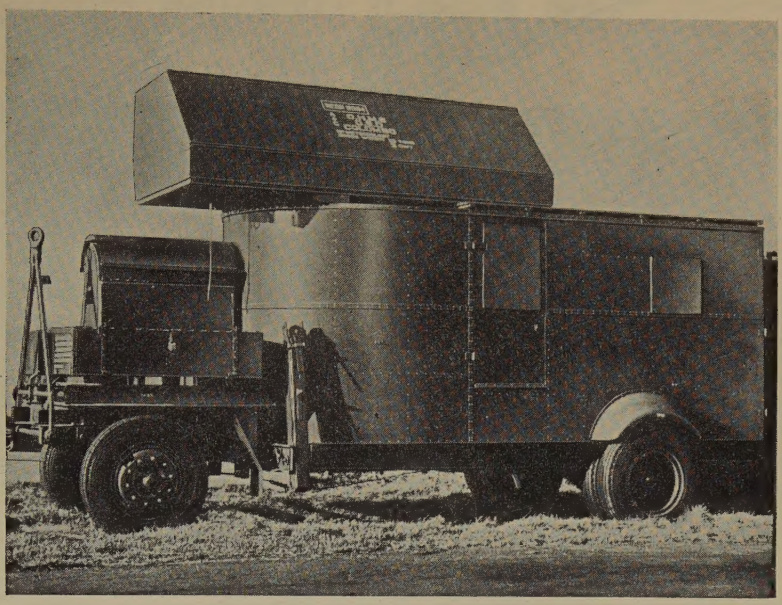

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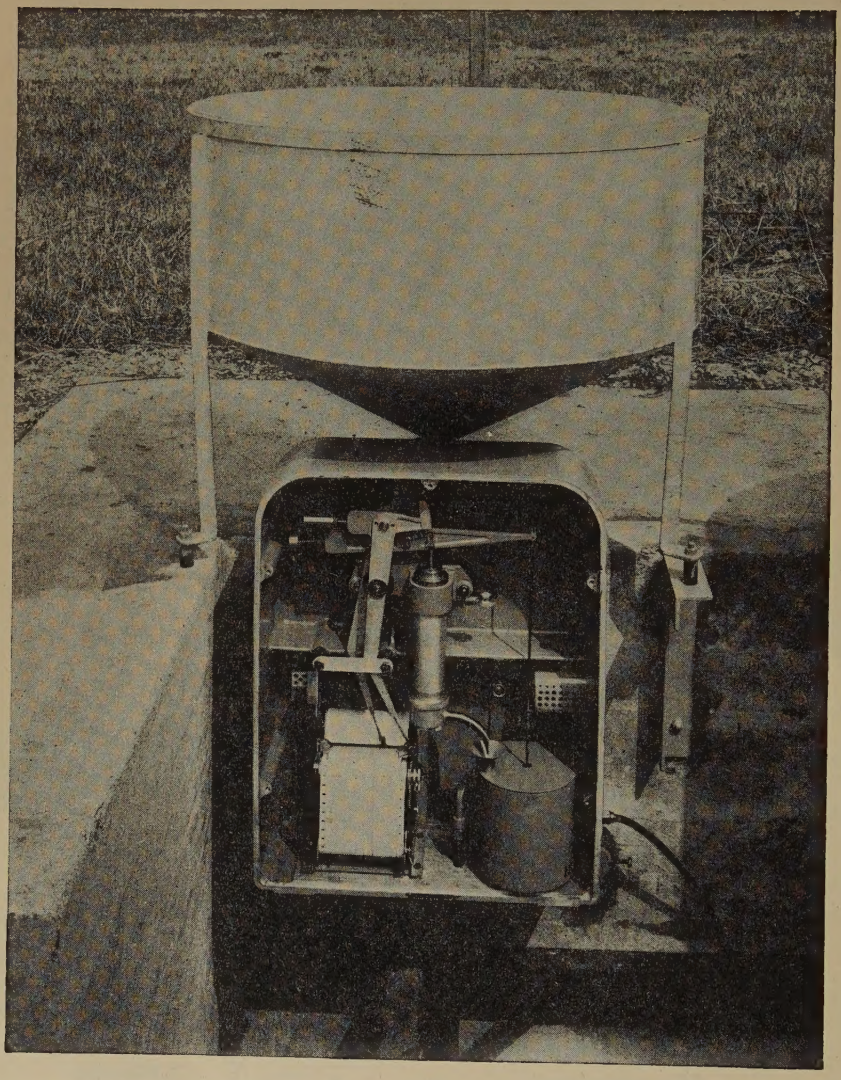

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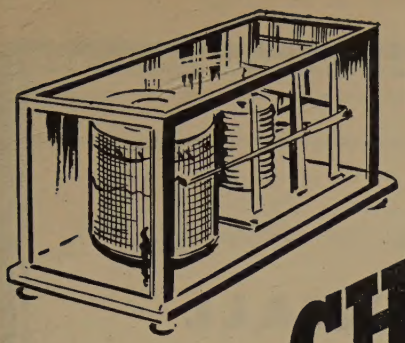

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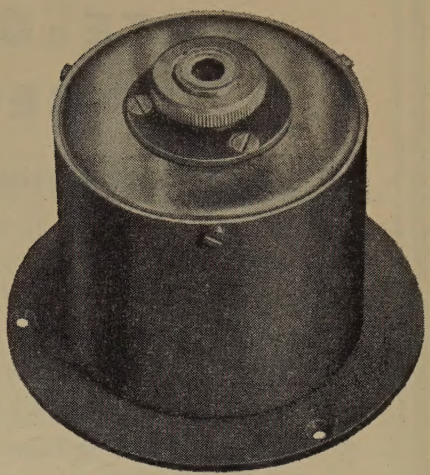

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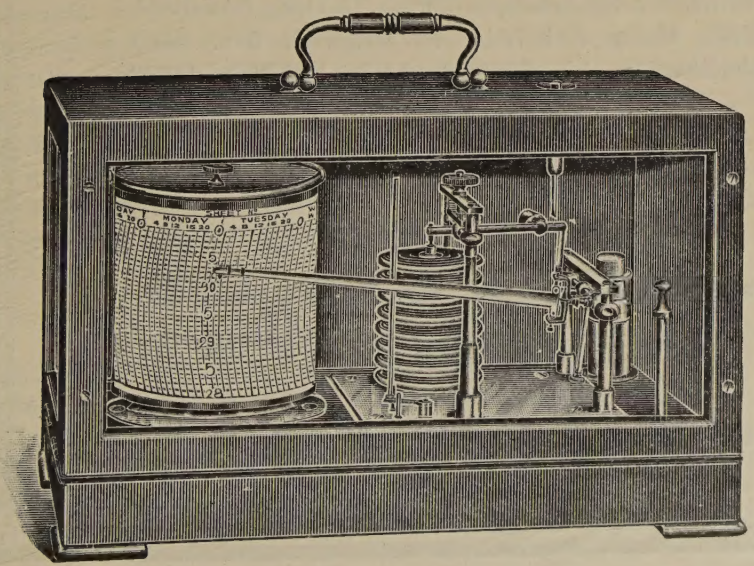

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Radio astronomy and the fringe of the atmosphere

By A. C. B. LOVELL, F.R.S.
Jodrell Bank Experimental Station, University of Manchester

Symons Memorial Lecture, delivered 27 April 1955

1. Introduction

Radio astronomy is concerned with the investigation of the solar system and the universe through two techniques. First, the purely receiving techniques working in the waveband from a few millimetres to about 20 metres, for the study of the radio waves emitted by the sun, the interstellar gas and the localized sources of radio emission in the universe (radio stars); secondly, the radio-echo techniques, akin to radar, involving the transmission and reception of radio waves in this waveband, for the study of meteors, the aurorae, and the moon. These recent developments have attracted considerable interest because of the many surprising astronomical discoveries which have been made. It is, perhaps, not generally appreciated that the incidental contributions to the study of the physics and meteorology of the high atmosphere and ionosphere have been considerable. These regions are now realised to be of great importance both because of the contemporary interest in their penetration by manned vehicles and by rockets, and because of the possible connections between conditions at these altitudes and more local meteorological conditions near the earth's surface. It is my privilege in this Symons Memorial Lecture to describe some of the recent work in radio astronomy which is concerned with this fringe of the earth's atmosphere. The results which will be presented are drawn from six lines of investigation. Firstly, in § 2 the scintillation of the radio stars, in relation to the information which it gives about the 400 km region of the atmosphere, will be described. The next three sections concern some aspects of the radio echo work on meteors in the 70 to 120 km region and deal with winds (§ 3), scale heights, densities and temperatures (§ 4), and diffusion rates (§ 5). The next section (§ 6) concerns the information about this region which is obtained from the radio echo work on the aurorae. Finally in § 7 the measurement of the total electron content in the earth-moon space by the study of lunar radio echoes will be discussed. It will not be possible in the scope of this paper to give details of the apparatus or techniques. These are covered in the original publications, and descriptions of the general principles may be found in several survey articles or books (for example, Ryle (1950), Pawsey and Bracewell (1955), Lovell and Clegg (1952), Lovell (1954a, 1954b, 1955)).

2. The scintillation of the radio stars

The phenomenon of fluctuations in the signal strength of the extra-terrestrial radio emission was first observed by Hey, Parsons and Phillips (1946). Subsequently, after the discovery that some of this radiation emanated from localized sources, it was believed that these variations represented actual fluctuations in strength of the signal emitted by the radio source. However, a series of spaced receiver measurements carried out jointly at Cambridge and Jodrell Bank (Smith, Little and Lovell 1950) showed that there was no correlation between the fluctuations observed at the two stations, implying that the

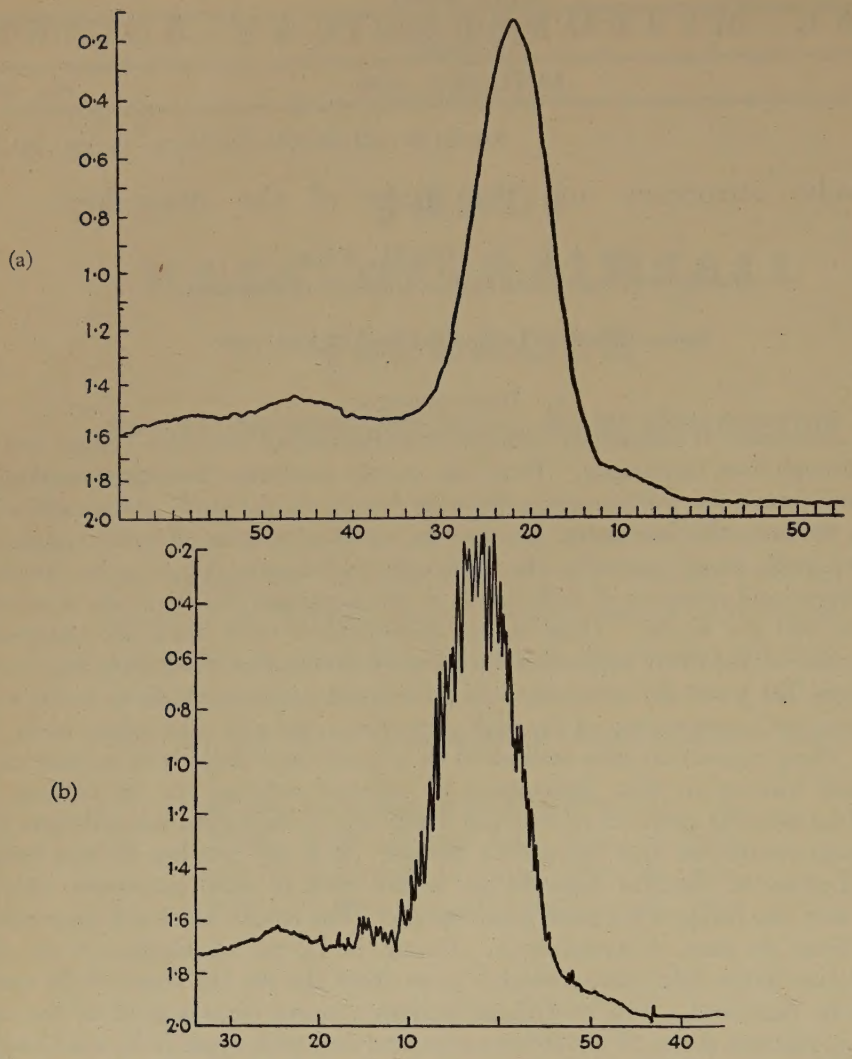

Figure 1. Records of the radio source in Cygnus from the Jodrell Bank 218-ft transit radio-telescope: (a) without fluctuations; (b) with fluctuations. The ordinates representing signal strength are in arbitrary units. The abscissae are in minutes of time.

source itself could not be responsible but that the fluctuations must originate either in the interstellar medium or in the earth's atmosphere. The effect has since been investigated in great detail by many workers (e.g., Little and Maxwell 1951, Ryle and Hewish 1950, Mills and Thomas 1951, Hewish 1952), and it is now known that the cause of the fluctuations is the presence in the F region at an altitude of about 400 km of irregular patches of ionization which act as a diffracting screen (Little 1951, Hewish 1951). This diffracting screen appears to be present on an average of one night in three in temperate latitudes and is correlated with the ' Spread F ' effect well known to ionospheric workers. The characteristic appearance of a radio star when observed on a narrow-beam radio telescope, with and without fluctuations, is shown in Figs. 1 (a) and (b). The primary cause of this diffracting screen is unknown; suggestions have been made that it is due to the accretion of interstellar material (Ryle and Hewish 1950), or to turbulence within the region (Mills and Thomas 1951, Maxwell 1954).

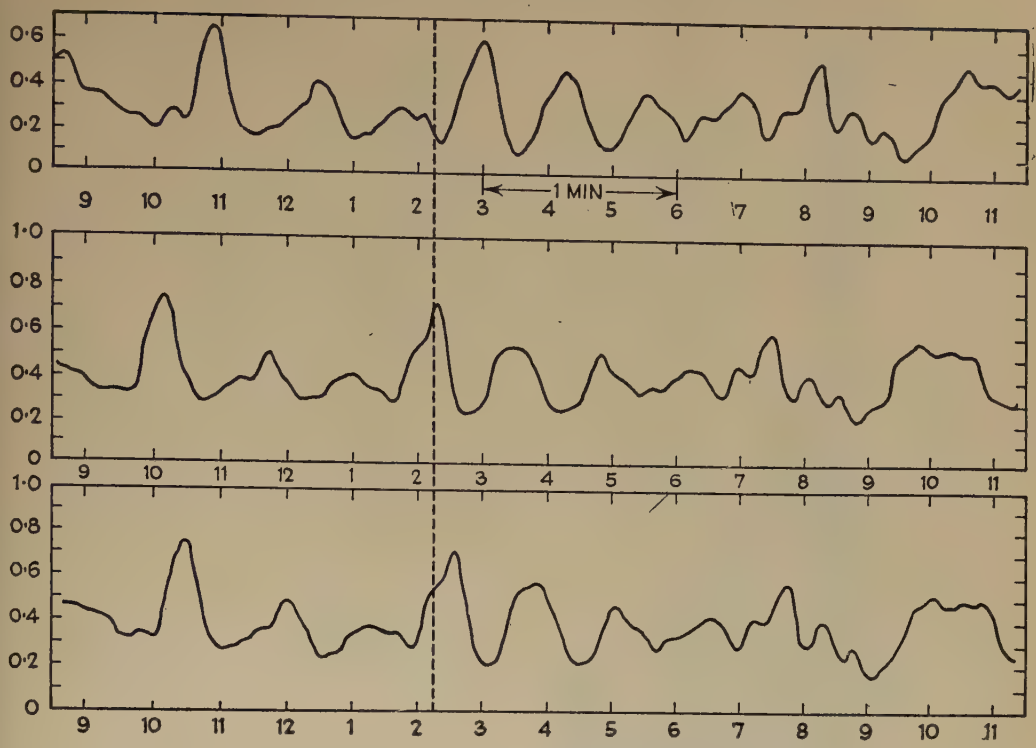

Figure 2. Radio-star scintillations from the source in Cassiopeia recorded simultaneously from receivers sited triangularly at spacings of approximately 1·2 km.

Our present interest in this effect arises from the discovery that the ionized clouds which constitute the diffracting screen are in rapid motion, and that, on the assumption that these clouds are isometric, the drift vector can be measured by simple triangulation (Maxwell and Little 1952). The apparatus consists essentially of three receivers spaced at the apices of a triangle, of sides such that the fluctuations observed at the three sites remain correlated (which is found experimentally to require spacings of less than about 3 km). A typical record obtained in this way is shown in Fig. 2. From the time displacement of the characteristic pattern, combined with a knowledge of the geographical positions of the receivers, it is possible to determine the drift vector. An indication of the results of such measurements is given in Figs. 3 and 4 (Maxwell and Dagg 1954). The distribution of drift speeds is shown in Fig. 3, from which it is seen that drifts up to 1,200 m/sec have been observed, with a most probable speed of 100 m/sec. Considerable detail exists regarding the variations from night to night of the average drift speed and its change throughout the night. Fig. 4 gives polar histograms of the drift directions, from which it is seen that before midnight the predominant tendency is for the drifts to be towards the west, whereas after midnight the prevailing direction is towards the east.

The precise interpretation of such records has recently been placed in some doubt because it seems that the original assumption of isometry of the electron clouds cannot be justified (Phillips and Spencer 1955, Spencer 1955). Recent experiments at Cambridge and Jodrell Bank show that the electron clouds are, in fact, elongated along the lines of magnetic force, the ellipticity being of the order of 10 to 1. In this case the triangulation technique would yield the component of drift velocity along the minor axis, and the

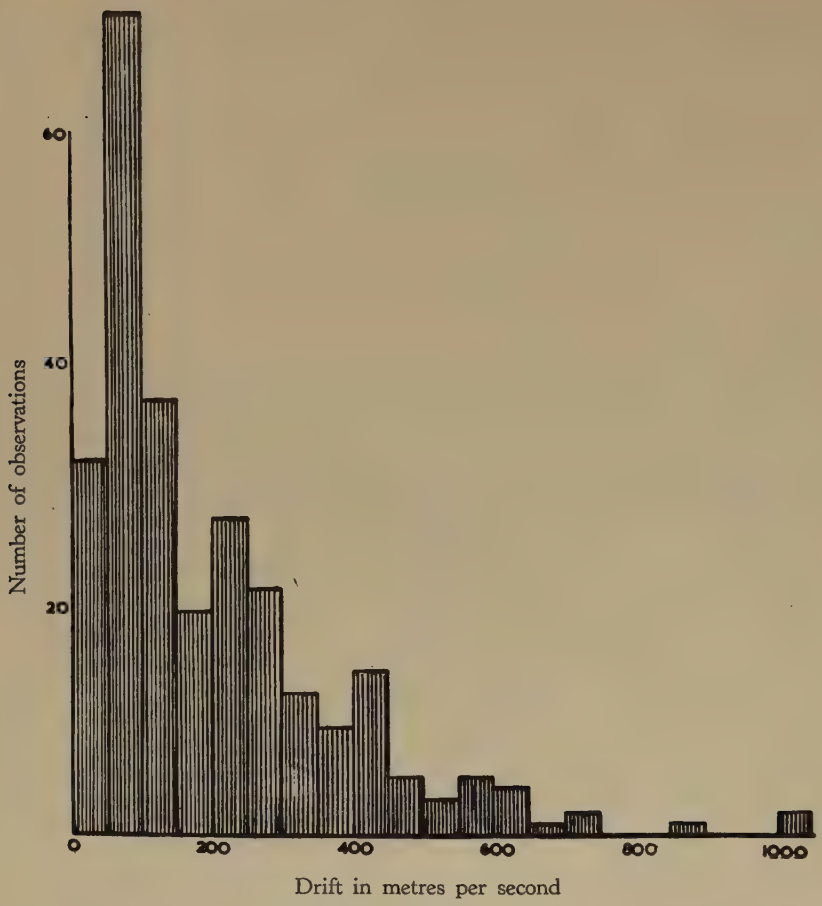

Figure 3. Histogram of *F*-region drift speeds determined from the radio-star scintillations.

predominance of E-W directions in Fig. 4 may be due to this cause. The extent of the N-S component remains to be determined, but the evidence leaves little doubt that the E-W component reverses in direction as shown in Fig. 4.

The above measurements refer to conditions in the *F* region overhead in latitude 50° to 60°N. Observations from latitude 52°N of the radio source in Cygnus at lower culmination give information about the drift conditions in the *F* region in the auroral zone, and the results differ considerably from those depicted in Figs. 3 and 4. In the first case scintillations are observed at all times, both by day and night, and the average drift speed is approximately 360 m/sec. Secondly, the reversal in drift direction occurs in the neighbourhood of 21 hr rather than 24 hr. Reference will be made in § 6 to a similar reversal found by radio-echo observations of the aurorae.

The rate of the fluctuations and the drift speeds are correlated with the disturbance variations in the geomagnetic field as shown in Fig. 5. This diagram refers to the region overhead. No precise figures are available for the auroral zone measurements but it is well known that the disturbance variation in the geomagnetic field is much greater in that zone and this is in accordance with the increased drift speeds and fluctuation rates found in the lower culmination measurements.

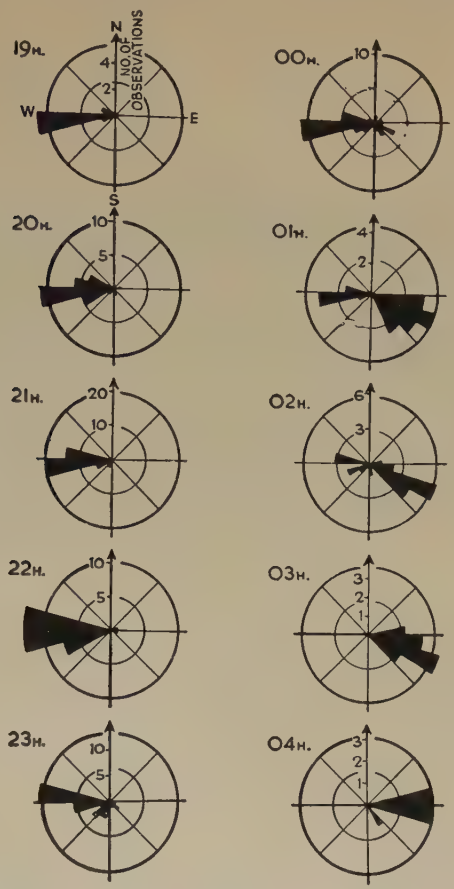

Figure 4. Hourly polar histograms of *F*-region drift directions determined by the radio-star scintillations (drift is *towards* directions indicated)

Throughout the past six years various methods have been developed for the investigation of drift motions throughout the ionosphere and although pulse sounding and meteor drift methods are well established in the study of the *E* and lower *F* regions (Briggs and Spencer 1955), this radio star technique is unique in the information which it gives about the upper *F* region. In the future, the extension of these measurements to other parts of the globe may be expected to assist in the study of the world morphology of the *F* region. In view, also, of the close correlation with geomagnetic activity, the widespread determination of upper *F* region drift velocities may provide criteria for an assessment of the dynamo theory of geomagnetic variations.

3. Radio echoes from meteors and the measurement of winds

Meteors enter the atmosphere with velocities ranging from about 11 km/sec to 72 km/sec. The majority of meteors normally observed as ' shooting stars ' in the night sky are completely evaporated in the 70 to 120 km region of the atmosphere. One result of the evaporation process is the creation of a dense trail of ionization which can be observed as a transient response on a radio echo equipment of the conventional radar type. The radio frequencies normally used lie in the range 30 to 80 mc/s, that is well

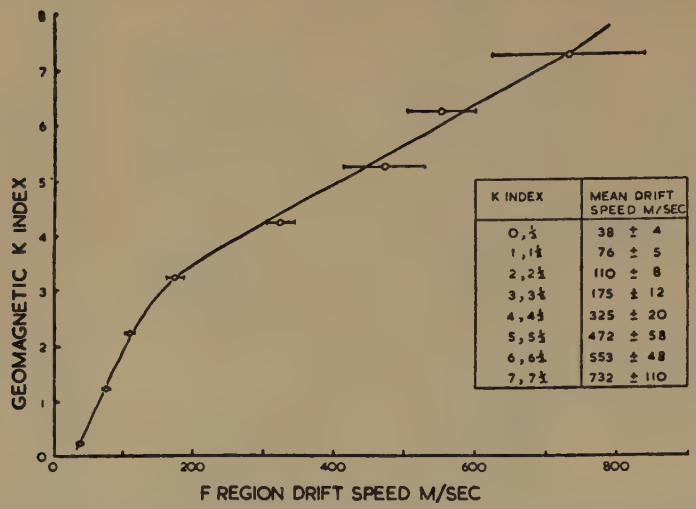

Figure 5. Correlation of *F*-region drift speed with geomagnetic disturbance variations: F-region drift speed versus geomagnetic *K* index.

above the critical frequency of the ionospheric regions. These techniques enable meteors to be studied by day or night, under any sky conditions. Precise methods have been developed for the measurement of velocities and orbits, and during recent years much progress has been made in the study of the astronomical aspects of meteors (see, for example, Lovell 1954a). Here, we are primarily concerned with the information about the high atmosphere which can be deduced from an investigation of the characteristics of these radio echoes.

The existence of high-velocity winds in the 80 to 100 km region of the atmosphere was first deduced from the observation of the movement of long-duration visual meteor trails and noctilucent clouds (Olivier 1942, Störmer 1937). These phenomena are rare and it was not possible to obtain systematic information about these wind movements. However, during the last few years radio-echo techniques have been developed which make possible the measurement of the drifts from short-duration trails. With echo rates which may amount to hundreds per hour, investigation of the wind structure at these altitudes is now possible. This work has been carried out at Stanford by Manning, Villard and Peterson (1950, 1952), in Adelaide by Robertson, Liddy and Elford (1953) and at Jodrell Bank by Greenhow (1950, 1952a, 1952b, 1954).

In the method used by Greenhow it is possible to determine the wind vector from about 30 minutes' observation under favourable conditions. A sample of the film record obtained is shown in Fig. 6. Four cathode ray tubes are photographed simultaneously for each meteor echo which trips the recording mechanism. A gives the range of the meteor on a conventional range-amplitude display. In B the time base is speeded up to separate the individual pulses, this gives the Fresnel diffraction pattern as the meteor crosses the aerial beam and enables the velocity of the meteor to be measured (Davies and Ellyett 1949). C and D record the phase variations as the ionized trails drift towards or away from the receiving station. A complete cycle in C and D represents a phase reversal caused by a drift of the trail by $\frac{1}{2}\lambda$. Since λ is 8 m, and the separation between each pulse is 0·006 sec it is clear that high drift speeds can be measured with considerable accuracy. The radial component of the wind velocity is obtained, and in order to determine the absolute wind speed and direction, a beamed aerial is directed alternately in two directions at right angles.

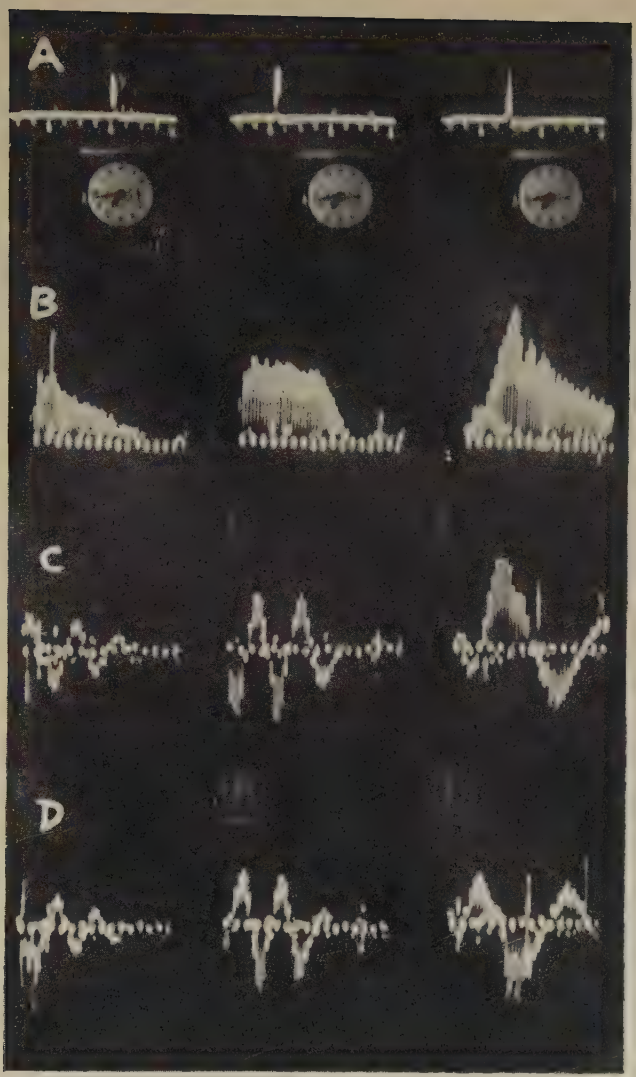

Figure 6. Three successive records of radio echoes from meteor trails as obtained in the wind-determination technique. In A the meteor echo is shown on a range time display, B shows the Fresnel diffraction pattern and C and D the phase changes due to the drift of the ionized trail.

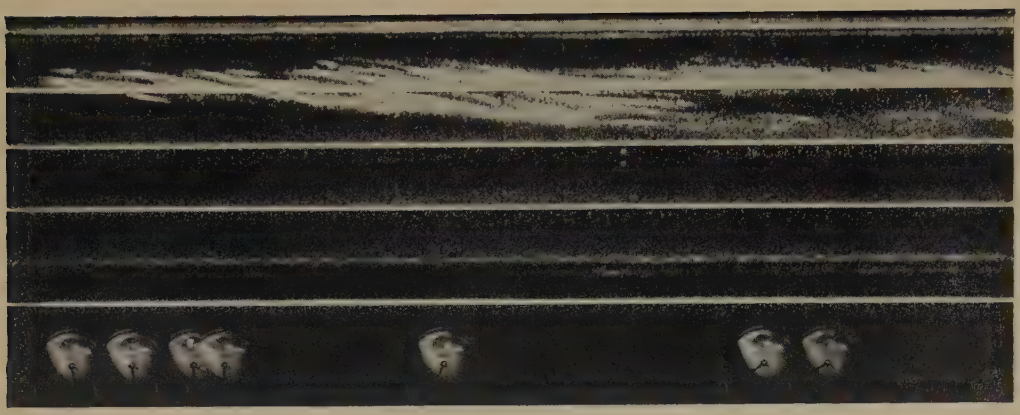

Figure 11. Photograph of a radio echo from an aurora of 27 April 1955. Time is horizontal and range vertical (the horizontal lines are spaced at 250 km range). The drift in range of the echo during the 15 min covered by the photograph is clearly visible. The double white spot is a radio echo from a meteor trail. (*See page 11*).

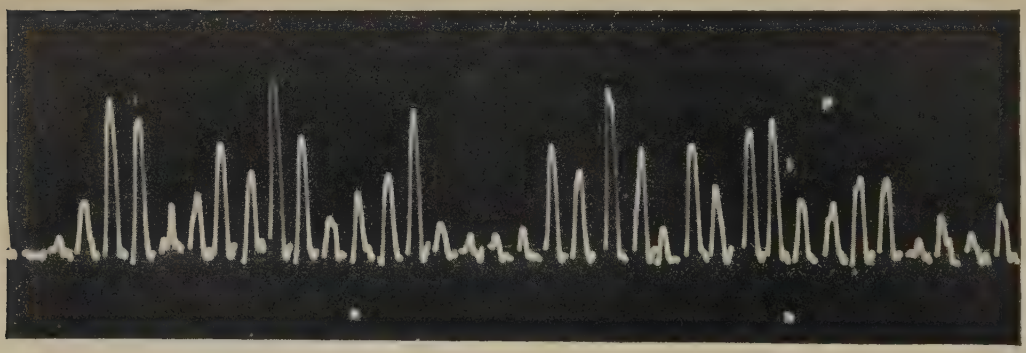

Figure 14. A section of a film record showing the rapid fading of the lunar radio echoes. The radio frequency was 120 mc/s, the pulse width 30 msec and the recurrence frequency 0·6/sec. The time base was strobed in this experiment so that only the section containing the successive lunar echoes was photographed. (*See page 12*).

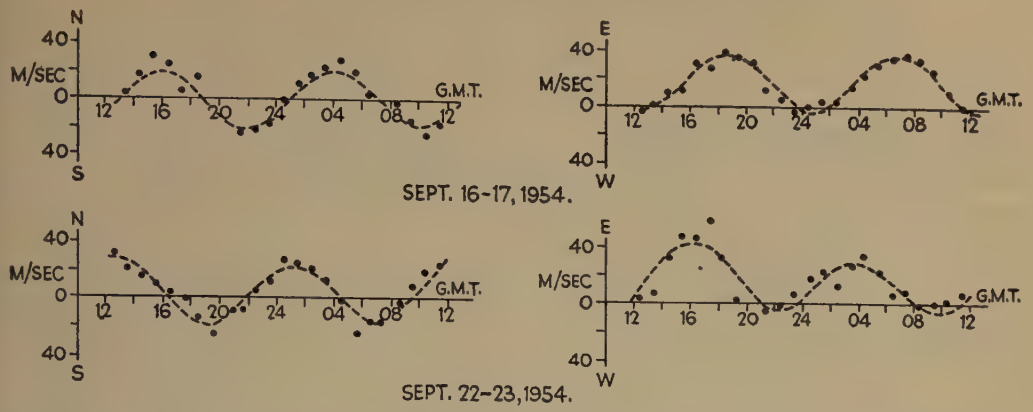

Figure 7. The N-S, E-W components of the wind at 95 km as determined by the radio-echo meteor technique for two days in September 1954.

Measurements have now been made for some years and a great deal of information has been collected about the diurnal and seasonal variation of the winds at these altitudes. Fig. 7 shows an example of the results for the N-S, E-W components of the wind velocity for two days in Sept. 1954. The wind reaches 40 m/sec and shows a semi-diurnal variation with a phase difference of 3 hr between the two components, indicating a wind vector rotating in a clockwise direction with a period of 12 hr. Similar results throughout the year show that the wind vector is towards the north at 06 hr and 18 hr except during the autumn months when the phase of the periodic wind component changes by 360°. Analysis of the 24-hr observations also shows that there is an appreciable prevailing wind. Fig. 8 shows a prevailing component towards the north during the winter and towards the south in summer. The prevailing E-W components are stronger and are towards the east in winter and summer and towards the west in spring and autumn.

The measurements are in general agreement with those obtained at Stanford by Manning, Villard and Peterson (1950, 1952). The results of the meteor work in Adelaide show similar components but with a rotation in the anti-clockwise direction – a result which is in agreement with the predictions of tidal theory. A comparison with the results obtained using ionospheric echo fading technique (Mitra 1949, Salzberg and Greenstone 1951, Phillips 1952) is also instructive. The daily periodic variations are not in evidence, although averages taken over several months show the presence of a similar 12 hr periodic wind. The drift speeds are greater than those given by the meteor methods, but since the precise altitude of the ionospheric measurement is unknown this may be a result of a difference in altitude of the measurements.

The meteor results refer to a mean height of 95 km. Further results are now being obtained on the variation with height. The periodic winds show large amplitude and phase variations, with increases in speed with height of about 3 m/sec per km. Techniques have also now been developed which will enable the degree of turbulence to be measured. The coordination of these measurements with the conditions which exist in the lower atmosphere presents an interesting problem in meteorology.

4. The measurement of scale heights and pressures

Until the recent use of rocket techniques for the exploration of the upper atmosphere, the photographic meteor techniques developed by Whipple (1943) and Jacchia (1952) provided almost the only source of precise data about densities in the 100 km region.

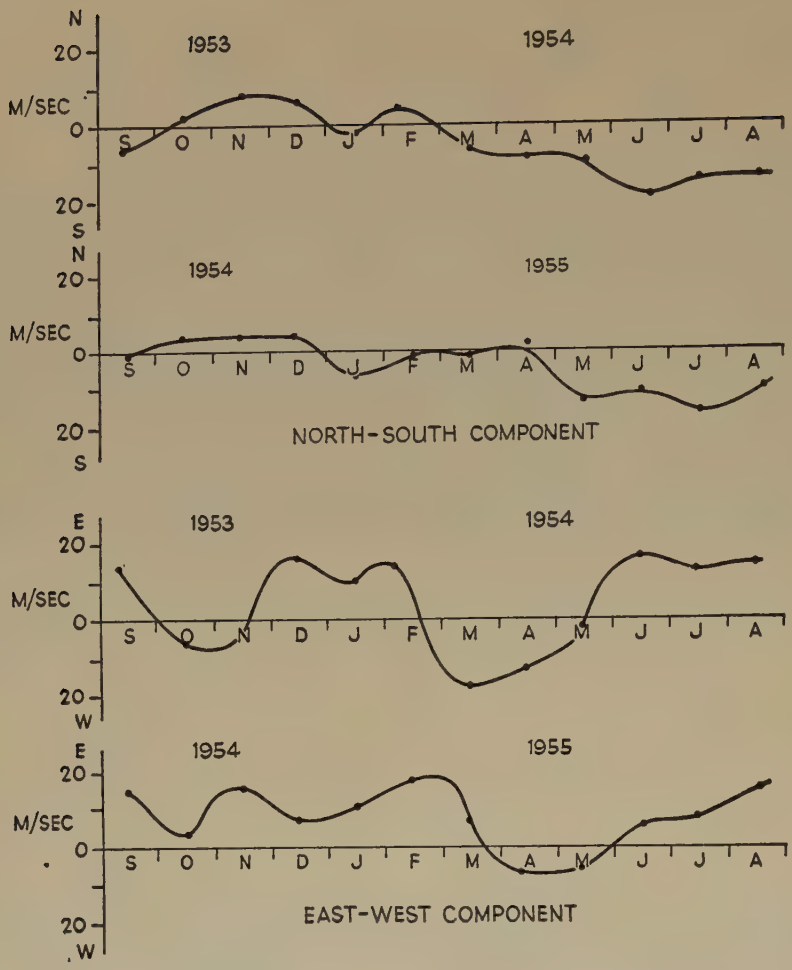

Figure 8. The variation of the prevailing wind at 95 km throughout the year as deduced from the radio-echo meteor observations.

During 1952-53 Kaiser and Evans developed a radio-echo meteor technique for such measurements (Kaiser 1954a, 1954b; Evans 1954), which possesses the advantages of simplicity compared with the rocket method, and of usefulness by day as well as by night compared with the meteor photographic methods. The radio method depends essentially on the measurement of the velocities (Davies and Ellyett 1949) and heights of the reflection points (Clegg and Davidson 1950) for large numbers of sporadic meteors. Height distributions are then plotted for a selection of meteors in given velocity groups. It can then be shown that the width of this height distribution depends only on the mass distribution of the incident meteors and on the atmospheric scale height. The general reason for this dependence can be readily appreciated. Firstly, for meteors moving into the atmosphere with a given velocity the larger meteors will have to intercept a greater air mass before complete evaporation occurs and can therefore be detected over a greater height range. Secondly, the extension in height for a given mass depends on the atmospheric pressure gradient (scale height). It is known from other work that the differential mass distribution for sporadic meteors can be represented by an inverse-power law with

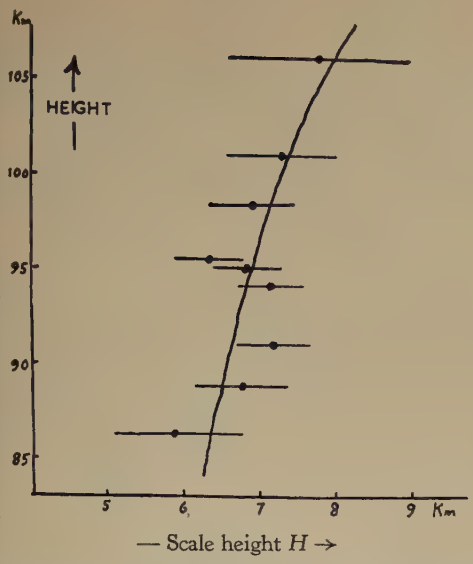

Figure 9. Scale heights as determined by the radio-echo meteor technique. The full line is the profile adopted by the rocket panel.

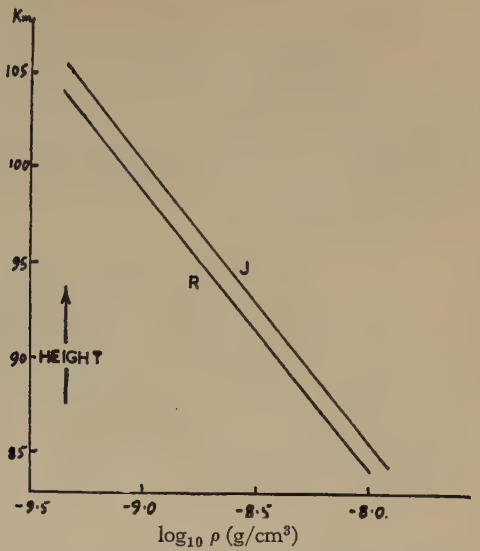

Figure 10. Atmospheric densities in the 80 to 100 km region as determined by the radio-echo meteor technique (J). The line R shows the densities measured in New Mexico by the rocket and photographic meteor techniques.

an exponent 2·00 ± 0·02. Kaiser (1954) has shown that in this case, if δh is the r.m.s. height deviation from the mean, and H the scale height then

$$\delta h = (1{\cdot}05 \pm 0{\cdot}02)\, H$$

Thus, by selecting different velocity groups of sporadic meteors the variation of H with altitude can be determined. The results obtained by Evans (1954, 1955) are shown in Fig. 9. The full line is the profile adopted by the Rocket Panel (1952) and the agreement between the two sets of measurements is very satisfactory.

The variation of pressure with height is determined by measuring the mean heights of the reflection points for meteors in different velocity groups. According to the theoretical work of Herlofson (1948) the atmospheric pressure p, at the height of maximum ionization (α electrons/cm path) of a meteor of velocity v is given by

$$p = \frac{2g\,Q}{v^2}\left(\tfrac{9}{4} H\alpha \cos^2 x\right)^{1/3}$$

where x is the zenith angle of the meteor trail, g is the gravitational acceleration, and Q is a parameter determined by the physical properties of the meteor. This equation cannot be used directly for the determination of p from individual meteors since the reflection point does not necessarily correspond to the point of maximum ionization along the trail. However, Kaiser (1954) has shown that in a given velocity group the height of maximum ionization of the smallest detectable meteor (α_m) (the ' characteristic height ') is related to the observed mean height of meteors in that velocity group. Since α_m can be computed from the parameters of the apparatus the equation can be used to determine p at the characteristic height. Evans (1954) has described the process in detail and Fig. 10 shows the results which he obtained by this method. For comparison with the rocket and meteor photographic techniques, the results are converted to densities, and it is seen that they are in very close agreement.

The apparatus involved in this work is relatively compact and easy to operate. The technique therefore opens up the prospect of establishing the extent of any diurnal or seasonal variation and of investigating any latitude dependence. The extension of the measurements to greater and lower heights requires many more measurements than have so far been made, and it seems possible that comparative radio echo and photographic programmes using Schmidt cameras may enable the lower height range to be extended to 50 or 60 km.

5. The measurement of diffusion coefficients

The ionized trail formed by the evaporation of a meteor as it enters the atmosphere disperses rapidly by diffusion. At the same time the amplitude of the radio echo received from the trail decays, generally within a fraction of a second, in the manner illustrated in the first and third photographic sets of Fig. 6 (line B). It can be shown (Herlofson 1948) that this decay is governed by the diffusion coefficient (D) and the wavelength (λ) according to the equation

$$A = A_0 \exp(-16\,\pi^2 Dt/\lambda^2)$$

where A_0 is the initial amplitude and A is the amplitude after time t. The decay constant, or duration to $1/e$ of the initial amplitude, is given by

$$T = \lambda^2/16\,\pi^2 D$$

and depends on λ and D only. In particular, it is independent of the line density of electrons in the trail. This applies only when the electron line density is less than 10^{12} electrons/cm. For higher values of line density the critical density may be exceeded at appreciable radii and the trail behaves as a totally reflecting cylinder. In these cases the echo duration depends on the line density as well as on λ and D. Hence for the measurement of D it is convenient to exclude these long duration radio echoes.

Examples of the measurement of D have been given by Greenhow and Neufeld (1955). The decay times can be measured with considerable accuracy from photographs similar to B in Fig. 6; and the height of the reflecting point is determined by the split-beam technique (Clegg and Davidson 1950, Evans 1954). Similar measurements using a continuous-wave technique have been made in Australia by Weiss (1955). Both sets of results agree over the height range 80 to 110 km. The value of D at 80 km is 10^4 cm^2/sec and at 100 km is 10^5 cm^2/sec. These diffusion coefficients are shown to be close to the theoretical estimates calculated from the rocket panel values of upper atmospheric air densities.

6. The investigation of the aurora borealis by the radio-echo technique

The possibility of investigating auroral phenomena by the kind of radio-echo techniques used for the meteor observations was demonstrated soon after the war (Lovell, Clegg and Ellyett 1947, Aspinall and Hawkins 1950), and in recent years this work has been greatly extended at Jodrell Bank (Bullough and Kaiser 1954, 1955), and in Canada (Forsyth 1953, Currie, Forsyth and Vawter 1953; McKinley and Millman 1953), in Norway (Harang and Landmark 1954), and in Sweden (Hellgren and Meos 1952). Many of the phenomena observed are complex and the radio-echo results are not fully understood in relation to the visual appearance of the aurorae. It seems clear, however, that in the near future these studies will become of great importance both for the studies of the auroral phenomena and of the conditions in the 100-km region of the atmosphere where the aurorae form.

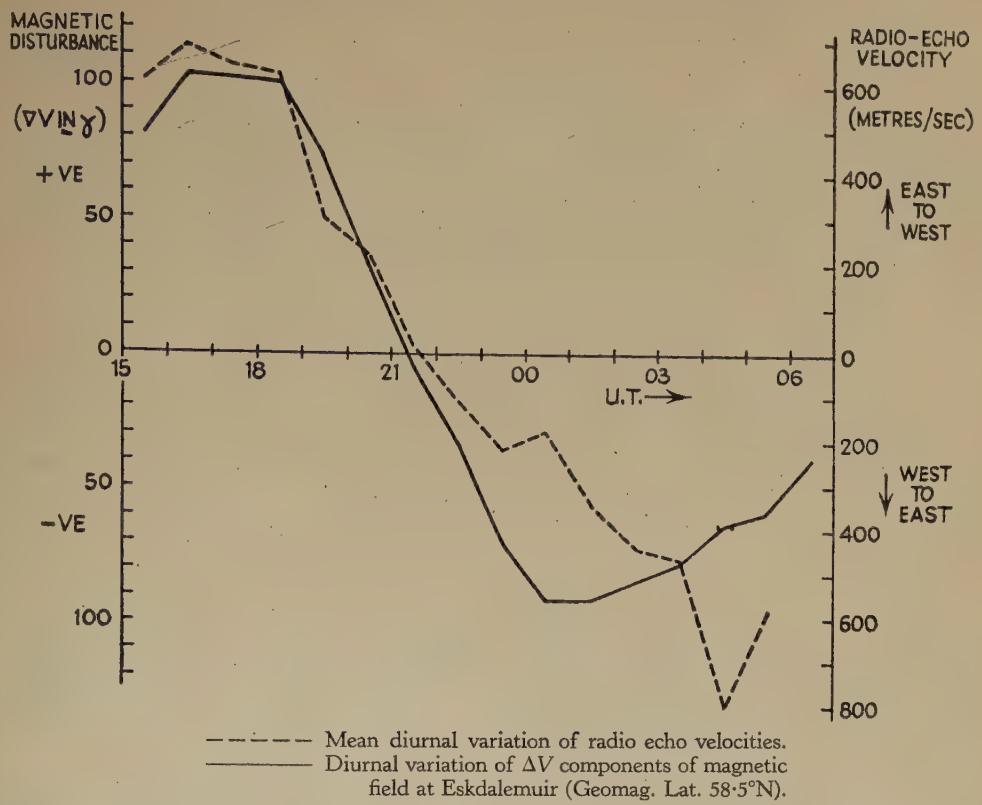

Figure 12. The diurnal variation in drift speeds as measured by the radio-echo technique compared with the variation in the ΔV magnetic component.

An example of these possiblities may be given with reference to Fig. 11 which shows a radio-echo response from an auroral formation on an intensity-modulated range-time display. One striking feature of this is the change in range of the echo with time. This is, in fact, a common observation and drifts up to 800 m/sec have been observed in this way. An analysis of the Jodrell Bank records obtained over several years indicates that these drifts have a marked diurnal variation as shown in Fig. 12. At 18 hr the drifts are receding with velocities of the order of 500 m/sec. A reversal occurs in the neighbourhood of 21 hr, and by 06 hr, the drifts are approaching with velocities of 500-600 m/sec. There are several interesting associations of this reversal. Firstly, the radio-echo auroral activity is closely correlated with terrestrial magnetic activity, and the diurnal variation in the ΔV component follows closely the drift curve of Fig. 12. The variations of ΔV measured at Eskdalemuir are plotted in Fig. 12 to show this correlation. Secondly, the reversal at 21 hr occurs at the same time as the reversal in the drifts measured by the radio star scintillation in the auroral zone as described in § 2. Finally, the rate of occurrence of the radio echoes from aurorae shows a minimum at this time, whereas the visual observations show a maximum. These latter variations are given in Fig. 13.

The explanation of these phenomena is unknown, although the drift motions are so high that it seems unlikely that they are due to winds in the sense that mass motion occurs. This belief is strengthened by the observation that E region winds measured as described in § 3 during aurorae show normal velocities not exceeding 50 m/sec. It is possible that the high auroral drifts may be an effect due to the redistribution of ionization.

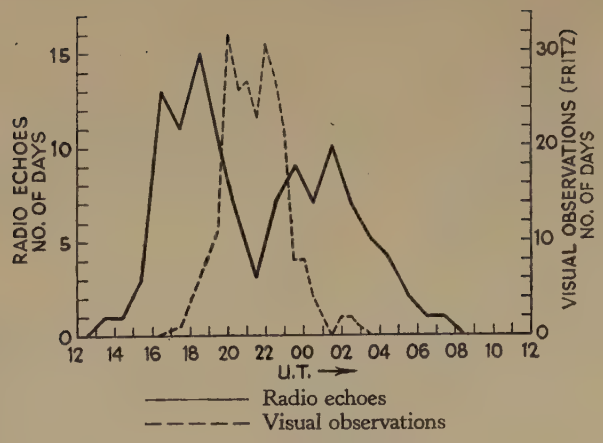

Figure 13. The diurnal variation in the rate of occurrence of aurorae as determined by the radio-echo technique, compared with the visual data.

7. Moon echoes and the ionosphere

The existing methods of ionospheric investigation give information about the electron density at particular levels but no reliable method has yet been devised which will give the total density throughout the ionosphere. This knowledge is, of course, vital to an understanding of the formation and behaviour of the ionospheric regions, and during the last few years a method using radio echoes from the moon has been devised at Jodrell Bank. A typical record of lunar radio echoes obtained on a frequency of 120 mc/s is shown in Fig. 14. The difference in amplitude of successive pulses (fast fading) has been shown to be due the libration of the moon. In addition there is a much slower period fading effect extending over periods of 20 to 30 min. Murray and Hargreaves (1954) investigated the reason for this slow period fading and were able to show that the effect was due to the rotation of the plane of polarization in the ionosphere (Faraday effect). The magneto-ionic theory shows that the polarization shift Ω is given by

$$\Omega = \frac{k}{f^2} \int N \, dr$$

complete rotations, where k is a factor including the geomagnetic field and the inclination of the geomagnetic vector to the line of sight, f is the radio frequency, $N(r)$ the electron number density/cm^3, and dr the element of path length. If the rotation occurs in a radio wave which has penetrated the entire ionosphere, as in the case of a moon echo, then $\int N \, dr$ is the total electron content of the ionosphere/cm^2 along the line of sight. The original experiments of Murray and Hargreaves were made at one frequency and gave only the rate of change of $\int N \, dr$. Subsequently the work was extended to two closely spaced frequencies and successful measurements have now been made of the total electron content (Browne, Evans, Murray and Hargreaves – in publication).

The technical difficulties are considerable and with the present aerial system measurements can be made only with the moon in transit for about 10 periods in each lunation. In the future it is hoped to use this method with the large steerable telescope at Jodrell Bank when these restrictions will be removed. The systematic data which can then be obtained are bound to be of considerable importance to our understanding of the ionosphere and of the whole field of solar-terrestrial relationships.

8. Conclusions

The above six examples of the investigation of the outer reaches of the atmosphere by radio astronomy have all been developed during the last ten years. At the same time the interest of meteorologists in these high-level phenomena, some of which may be directly associated with local terrestrial conditions, has greatly increased. The results of future cooperation between meteorologists and radio astronomers in the extension and interpretation of this work will be awaited with great interest.

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Energy changes involved in disturbing a dry atmosphere

By F. K. BALL

Division of Meteorological Physics, Commonwealth Scientific and Industrial Research Organization, Victoria, Australia

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SUMMARY

A method is presented for calculating the energy changes involved in disturbing a layer of a dry atmosphere. The special case of complete mixing is considered in detail and it is shown that this can only occur spontaneously when the Richardson number is less than one-half. The kinetic energy absorbed or released when a layer of any thickness and lapse rate is completely mixed can be estimated from a graph. A second graph gives the kinetic energy necessary to maintain a given temperature gradient when heat is added to or withdrawn from any given level within the layer. Application to frost prevention shows that the energy required to maintain mixing in a 100-ft layer while heat is being lost at the base is only a small fraction of the input energy from the normal design of frost fan.

1. INTRODUCTION

An understanding of the processes of generation and destruction of kinetic energy in the atmosphere is one of the central problems of meteorology. This paper will be largely concerned with one particular aspect of this general problem, namely the kinetic energy changes caused by mixing a dry horizontally uniform atmospheric layer of known initial temperature distribution. This type of problem arises in connection with frost-fan operation, where an established inversion is mixed by the fan; furthermore, during the subsequent course of operation of the fan, the air is cooled continuously by transfer of heat to the ground and the fan will redistribute this cooling throughout the mixed layer. It is intuitively obvious that some of the kinetic energy generated by the fan will be absorbed by buoyancy forces, firstly in achieving the mixed state and secondly in maintaining it. Another example of this type of problem is the release of kinetic energy by the mixing of an unstable layer in the atmosphere.

The solution has previously been approached along two apparently quite different lines : the discontinuous approach, originated by Margules (1910), where a layer of the atmosphere is overturned and the kinetic energy change calculated from the difference between initial and final enthalpy, and the continuous approach of Richardson (1920) where the rate of production of turbulent kinetic energy is found in terms of the wind shear and temperature gradient in the layer. Margules's process is both reversible and adiabatic, whereas the mixing processes which occur either naturally in the atmosphere or during frost-fan operation are clearly irreversible and usually non-adiabatic. Furthermore, it is not clear how Richardson's method can be applied to the processes involved in the operation of a frost fan. It therefore seems desirable to formulate a more general theory which will apply to these problems and others of similar type. Margules's discontinuous approach will be adopted here though it will be seen later that it is closely related to Richardson's method.

Consider a layer of a dry atmosphere with a known initial temperature distribution. Suppose the layer is adiabatically disturbed in such a manner that it remains horizontally uniform. If the initial temperature distribution is stable, then work must be done against buoyancy forces during the course of the disturbance. This work is provided by the destruction of turbulent kinetic energy – energy generated by the frost fan in the case of artificial mixing and by wind shear in the free atmosphere. If the initial temperature distribution is unstable then kinetic energy is liberated by buoyancy forces when mixing

takes place. It is desirable to know precisely how much kinetic energy is absorbed or released when a given degree of mixing occurs. It is well known that the energy required to disturb a layer adiabatically is given by the increase in enthalpy caused by the disturbance (Margules 1910). In general, however, though the form of the final temperature distribution may be known, the actual temperature level is not, and some additional condition must be invoked. For example, if a layer is completely mixed the resulting temperature gradient is dry adiabatic, but the enthalpy change is indeterminate without further knowledge or assumption. It is for this reason that the problem cannot be treated in general by ordinary methods on aerological diagrams.

Mixing occurs in an unstable layer of a horizontally uniform atmosphere by the movement of buoyant bubbles of air or by the operation of thermal columns. The kinetic energy of these features is the energy released by the mixing process. Their scale is limited by the thickness of the unstable layer and consequently the energy released by this type of instability appears as convective turbulence, modified by any wind shear which may exist in the layer, and the maximum size of the turbulent elements is of the same order of magnitude as the thickness of the layer.

Notation in addition to generally accepted symbols

A = large horizontal area.
E = total kinetic energy per unit horizontal area of the disturbed layer.
F = upward transfer of potential heat per unit area during the disturbance.
G = upward transfer of mean flow kinetic energy per unit area during the disturbance.
G^* = upward transfer of turbulent kinetic energy per unit area during the disturbance.
H = upward transfer of enthalpy per unit area during the disturbance.
K = mean flow kinetic energy per unit mass.
K^* = turbulent kinetic energy per unit mass.
M = downward transfer of momentum per unit area during the disturbance.
m = suffix indicating the value after the disturbance.
P = hydrostatic pressure.
P_0 = standard pressure.
P_1 = hydrostatic pressure at the base of the disturbed layer.
P_2 = hydrostatic pressure at the top of the disturbed layer.
P_3 = hydrostatic pressure at which heat is added.
p = total pressure.
$p^* = p - P$.
$\delta P = P_1 - P_2$.
$\overline{P} = \frac{1}{2}(P_1 + P_2)$.
$P' = P - \overline{P}$.
Q = quantity of heat added per unit area.
q = quantity of heat added per unit mass.
t = duration of disturbance.
U = horizontal velocity meaned over the area A.
w = vertical velocity relative to the isobaric surface.
W^* = work done by pressure fluctuations.
$x = P_3/P_1$.
$y = P_2/P_1$.
α = fractional change in gradient.
$\beta = -\partial T/\partial z$.
$\gamma = C_p/C_v$ = ratio of specific heats.
Δ = increase during the course of the disturbance.

2. Fundamental Equations

Following Richardson, (1920) attention will be confined to a large horizontal area of the atmosphere A, bounded by vertical walls across which no net transfer occurs. The disturbance takes place between two isobaric surfaces so that the mass of the layer remains constant. It is supposed that the process is adiabatic, though not necessarily isentropic; the effect of relaxing this assumption will be discussed presently. Margules proved in 1906 that the kinetic energy released or absorbed by an adiabatic disturbance is given by the decrease or increase of enthalpy caused by the disturbance. If the pressures at the lower and upper boundaries of the layer are P_1 and P_2 respectively, the mean flow and turbulent kinetic energies per unit mass are K and K^* respectively, and Δ signifies the change which takes place during the disturbance, then the change in total kinetic energy per unit horizontal area is given by :

$$\Delta E = \int_{P_2}^{P_1} \frac{\Delta (K + K^*)}{g} dP = - \int_{P_2}^{P_1} \frac{C_p \Delta T}{g} dP \quad . \quad . \quad (1)$$

When there is no kinetic energy present initially a disturbance can only take place spontaneously if it is accompanied by a decrease in enthalpy. The equation gives no information about the form in which the kinetic energy appears; distinction between changes in K and K^* can only be made by the application of principles in addition to that of total energy conservation.

Eq. (1) can easily be generalized to include non-adiabatic disturbances. Suppose a quantity of heat q per unit mass is added at the level P then Eq. (1) is modified to :

$$\Delta E = \int_{P_2}^{P_1} \left(\frac{q - C_p \Delta T}{g} \right) dP \quad . \quad . \quad . \quad (2)$$

and in the absence of initial kinetic energy the criterion for the occurrence of a disturbance becomes :

$$\int_{P_2}^{P_1} \frac{q}{g} dP > \int_{P_2}^{P_1} \frac{C_p \Delta T}{g} dP \quad . \quad . \quad . \quad (3)$$

as stated by Normand (1946).

If q and ΔT are known by direct measurement, then Eq. (2) is sufficient to determine the change in kinetic energy. In practice however ΔT and q can never be measured with sufficient accuracy for the change in kinetic energy to be calculated, since increases in temperature at one point are usually largely compensated by decreases at another and the change in enthalpy consequently depends on the difference between nearly equal quantities. Furthermore the heat added is mainly consumed in increasing the enthalpy and the calculation of the kinetic energy change involves the further difference of two nearly equal quantities neither of which is known with sufficient accuracy.

The preceding relationships are merely statements of energy conservation and give no information whatever concerning the magnitudes of the temperature changes which can occur in atmospheric mixing. There are however two further relationships which impose a considerable restriction on the possible distribution of temperature change. The first of these to be considered here expresses the conservation of potential heat. It is convenient to introduce the concept of potential heat since it enables many of the results that follow to be expressed in a form which is both concise and easily understood. The potential heat of an element is defined as the enthalpy that it would have if brought adiabatically to standard pressure, P_0 say, and consequently bears the same relation to potential temperature as enthalpy does to temperature. Potential heat was defined by Helmholtz in 1888. The extensive quantity, *waermegehalt* as Helmholtz called it, was

almost immediately dropped in favour of the intensive quantity potential temperature, advocated by Bezold.

By definition, potential heat is a conservative property for the adiabatic motion of an element. Two elements of different potential temperature which mix at some level in the atmosphere also conserve their potential heat. It is to be expected therefore that if molecular effects are neglected, the total potential heat of a layer is unchanged by an adiabatic disturbance. This result may be written :

$$\int_{P_2}^{P_1} \frac{C_p \, \Delta\theta}{g} \, dP = 0 \quad . \quad . \quad . \quad . \quad (4)$$

where θ is the potential temperature. It can be seen that increases in potential temperature are precisely compensated by decreases at other levels within a disturbed layer. The changes in temperature will also approximately compensate each other, so that, as one would expect, adiabatic mixing serves largely to redistribute heat and cannot cause a large overall change of temperature.

Eq. (4) does not in any way restrict the magnitude of the potential temperature change which can occur at a particular level. There remains the possibility of the mixing process redistributing the potential heat in such a manner that a very large increase in potential temperature occurs in one half of the layer and a very large decrease in the other. This possibility is excluded by the law of increasing entropy, viz.

$$\int_{P_2}^{P_1} \frac{C_p}{g} \, \Delta \log \theta \, dP \geqslant 0. \quad . \quad . \quad . \quad . \quad (5)$$

whereas Eq. (4) states that adiabatic mixing merely redistributes potential heat, the inequality (5) implies further that mixing tends to reduce potential temperature contrasts.

In the non-adiabatic case a quantity of heat q added at the level P is equivalent to the addition of a quantity of potential heat $q \left(\frac{P_0}{P}\right)^{\frac{\gamma-1}{\gamma}}$ and to an entropy increase of q/T so that Eq. (4) is modified to :

$$\int_{P_2}^{P_1} \frac{C_p \, \Delta\theta}{g} \, dP = \int_{P_2}^{P_1} \frac{q}{g} \left(\frac{P_0}{P}\right)^{\frac{\gamma-1}{\gamma}} dP \quad . \quad . \quad . \quad (6)$$

and the inequality (5) becomes :

$$\int_{P_2}^{P_1} \frac{C_p \, \Delta \log \theta}{g} \, dP \geqslant \int_{P_2}^{P_1} \frac{q}{Tg} \, dP \quad . \quad . \quad . \quad (7)$$

3. Heat transfer

Attention has so far been directed towards the disturbed layer as a whole. In contrast, the energy balance at a particular level within the layer will now be considered. In this case it is necessary to take account of the addition and removal of both heat and kinetic energy to or from the level by the mixing process. If the total upward transfers per unit area, of heat, mean flow kinetic energy, and turbulent kinetic energy, during the course of the disturbance are denoted by H, G and G^* respectively, then the energy balance equation is :

$$\Delta (K + K^* + C_p \, T) = g \frac{\partial}{\partial P} (G + G^* + H) + q \quad . \quad . \quad (8)$$

Eq. (2) can be derived immediately from Eq. (8) by integrating throughout the whole disturbed layer outside which G, G^* and H vanish by definition. Eq. (8) may be converted to other more useful forms by applying the principles of conservation of potential heat and momentum.

Consider first the potential heat balance at a particular level. If the total upward transfer of potential heat per unit area is F, then neglecting molecular effects, conservation of potential heat gives :

$$C_p \Delta\theta = g \frac{\partial F}{\partial P} + q \left(\frac{P_0}{P}\right)^{\frac{\gamma-1}{\gamma}} \quad . \quad . \quad . \quad (9)$$

and

$$F = \frac{1}{A} \iint C_p \rho w \theta \, dA \, dt$$

where the integration extends over the large horizontal area A and the time of the disturbance t, w being the vertical velocity relative to the isobaric surface. Similarly the heat transfer is :

$$H = \frac{1}{A} \iint C_p \rho w T \, dA \, dt$$

In order to find an enthalpy balance equation similar to the potential heat balance Eq. (9), it is necessary to determine the relation between heat transfer and potential heat transfer. This can be done as follows :

$$T = \theta \left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}} \left(1 + \frac{p^*}{P}\right)^{\frac{\gamma-1}{\gamma}}$$

where p^* is the fluctuation of the pressure from its hydrostatic value P during the course of the disturbance. Expanding by the binomial theorem, neglecting powers of p^*/P greater than the first and substituting in the expression for H, gives the desired relation :

$$H = F \left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}} + \frac{1}{A} \iint p^* w \, dA \, dt \quad . \quad . \quad . \quad (10)$$

The last term on the right-hand side represents the work done by the pressure fluctuations and will be denoted by W^*.

An enthalpy balance equation can now be deduced from Eqs. (9) and (10).

$$C_p \Delta T = g \frac{\partial}{\partial P} (H - W^*) + q - \frac{gF}{C_p \rho \theta} \quad . \quad . \quad (11)$$

The change in temperature is therefore not only caused by heat transfer into the level and heat absorbed at the level but also by the potential heat transfer through the level. This last term is a transformation term representing the rate of transformation of heat into kinetic energy. This can readily be seen by substituting for ΔT in Eq. (8) using Eq. (11), giving

$$\Delta (K + K^*) = g \frac{\partial}{\partial P} (G + G^* + W^*) + \frac{gF}{C_p \rho \theta} \quad . \quad . \quad (12)$$

The significance of $gF/C_p \rho\theta$ as a transformation term is now clear since it represents a loss of heat in Eq. (11) and a gain in kinetic energy in Eq. (12). It follows that, in the conditions envisaged in this paper, generation of kinetic energy is always accompanied by the upward transfer of potential heat and destruction of kinetic energy by the downward transfer of potential heat.

On integrating Eq. (12) throughout the whole disturbed layer, whence G, G^* and W^* vanish at the upper and lower boundaries, a further equation for the total generation of kinetic energy by the disturbance is obtained :

$$\Delta E = \int_{P_2}^{P_1} \frac{F}{C_p \rho \theta} \, dP \quad . \quad . \quad . \quad . \quad (13)$$

This equation is valid for non-adiabatic disturbances and it is not necessary to neglect H at the boundaries. It is much more convenient than Eq. (2) for calculating kinetic energy changes in certain types of problems, notably those in which the kinetic energy is generated or absorbed by transfer of heat into or through the disturbed layer. The energy required to maintain the completely mixed state when a layer is subject to cooling from below is an example of this type of problem.

In all the preceding equations there has been nothing to distinguish between the two forms of kinetic energy. This distinction is made by applying conservation of momentum to the layer. If the mean velocity at a particular level before the disturbance is U and the downward transfer of horizontal momentum is M then :

$$\Delta U = -g\frac{\partial M}{\partial P} \quad . \quad . \quad . \quad . \quad (14)$$

Furthermore, using the suffix m to denote values after the disturbance, the increase in mean flow kinetic energy is :

$$\begin{aligned}\Delta K = & \tfrac{1}{2}(U_m^2 - U^2)\\ &= -\tfrac{1}{2}(U_m + U)\, g\frac{\partial M}{\partial P}\\ &= -g\frac{\partial}{\partial P}\left[\tfrac{1}{2}(U_m + U)\, M\right] + Mg\frac{\partial}{\partial P}\tfrac{1}{2}(U_m + U)\end{aligned}$$

The expression $-\frac{1}{2}(U_m + U)\, M$ is the product of the momentum transfer and the mean velocity and represents the transfer of mean flow kinetic energy previously denoted by G. The balance equation for mean flow kinetic energy can now be written :

$$\Delta K = g\frac{\partial G}{\partial P} + Mg\frac{\partial}{\partial P}\tfrac{1}{2}(U_m + U) \quad . \quad . \quad . \quad (15)$$

The first term on the right-hand side of this equation merely represents the redistribution of mean flow kinetic energy within the system. The last term is a transformation term and since M and $\frac{\partial}{\partial P}\frac{1}{2}(U_m + U)$ always have opposite signs it indicates that the mean-flow kinetic energy is decreased when the momentum of a layer is redistributed by mixing. As pointed out by Swinbank (1952) Richardson's analysis also implies that the mean-flow kinetic energy is continuously decreased.

It is for this reason, and the fact that any attempt at separation of ΔK and ΔK^* tends to give rise to imponderable terms, that extreme care must be exercised in making practical applications of Richardson's criterion or the adaptations thereof. In practice, the mean-flow kinetic energy must increase about as often as it decreases, and its source must lie in processes not considered here, nor by Richardson. It is reasonable to identify this source as residing in lack of horizontal uniformity, which in turn has its origin in heat transfer.

Substitution from Eq. (15) in Eq. (12) gives the balance equation for turbulent kinetic energy :

$$\Delta K^* = g\frac{\partial}{\partial P}(G^* + W^*) + \frac{gF}{C_p\,\rho\theta} - Mg\frac{\partial}{\partial P}\tfrac{1}{2}(U_m + U) \quad . \quad (16)$$

This is a finite difference form of Richardson's expression for the generation of turbulent kinetic energy, with additional terms of similar character to those introduced by Calder (1949). The derivation is however quite different from Calder's since he makes no use of total energy balance or potential heat balance and derives his final result by approximations that are open to some doubt (Priestley and Sheppard 1952). The second term gives

the generation of turbulent kinetic energy by buoyancy and the third by shearing stress. The two parts of the first term are in the nature of boundary effects and little is known about their order of magnitude. As mentioned previously, they represent transfer of turbulent kinetic energy and work done by pressure fluctuations. If Eq. (16) is integrated throughout the disturbed layer, supposing that at the boundaries G^* and W^* are negligible compared with the total change in turbulent kinetic energy, then the following result is obtained :

$$\int_{P_2}^{P_1} \frac{\Delta K^*}{g}\, dP = \int_{P_2}^{P_1} \left[\frac{F}{C_p \rho \theta} - M \frac{\partial}{\partial P} \tfrac{1}{2} (U_m + U) \right] dP \quad . \quad . \quad (17)$$

Whereas Eq. (1) applies only to the adiabatic disturbance of a strictly isolated layer, Eq. (17) applies to a non-adiabatic disturbance allowing for transfer of both heat and mean flow kinetic energy across the isobaric boundaries.

4. The thin layer

If the pressure difference between the top and the bottom of the disturbed layer is small compared with the mean pressure of the layer, then expressions for energy changes can readily be derived in a much more tractable and physically revealing form. The thin layer will therefore be considered here in detail. Let $P_1 - P_2$ equal δP and $\overline{P}$ equal $\frac{1}{2}(P_1 + P_2)$. If X is some property of the fluid let $\overline{X}$ be defined as follows :

$$\overline{X} = \frac{1}{\delta P} \int_{P_2}^{P_1} X\, dP$$

and

$$X' = X - \overline{X}.$$

Using this notation, Eqs. (1) and (4) for an adiabatic disturbance can be written :

$$\Delta (\overline{K + K^*}) = - C_p \Delta \overline{T}$$

$$\Delta \overline{\theta} = 0$$

On putting T equal to $\theta \left(\frac{\overline{P}}{P_0} \right)^{\frac{\gamma-1}{\gamma}} \left(1 + \frac{P'}{\overline{P}} \right)^{\frac{\gamma-1}{\gamma}}$ and neglecting powers of $P'/\overline{P}$ greater than the first, algebraic manipulation yields the following result :

$$\Delta (\overline{K + K^*}) = R \left(\frac{\overline{P}}{P_0} \right)^{\frac{\gamma-1}{\gamma}} \left(\frac{\overline{\theta' P'} - \overline{\theta_m' P'}}{\overline{P}} \right) \quad . \quad . \quad (18)$$

Furthermore $$\Delta \overline{K} = \tfrac{1}{2} (\overline{U_m^2} - \overline{U^2})$$

and $$\Delta \overline{U} = 0$$

Therefore $$\Delta \overline{K} = \tfrac{1}{2} (\overline{U_m'^2} - \overline{U'^2}) \quad . \quad . \quad . \quad . \quad (19)$$

The increase of entropy caused by the mixing is given by :

$$C_p \Delta \overline{\log \theta} = \frac{C_p}{2} \left(\frac{\overline{\theta'^2} - \overline{\theta_m'^2}}{\overline{\theta}^2} \right) \quad . \quad . \quad . \quad (20)$$

provided θ' is small compared with $\overline{\theta}$.

The preceding results take a simple form when the gradients of velocity and potential temperature can be regarded as constant within the layer. We then have :

$$\theta' = P' \frac{\partial \theta}{\partial P}$$

$$U' = P' \frac{\partial U}{\partial P}$$

$$\overline{P'^2} = \frac{(\delta P)^2}{12}$$

Under these circumstances Eq. (20) reduces to :

$$C_p \, \Delta \overline{\log \theta} = \frac{C_p (\delta P)^2}{24 \, \overline{\theta}^2} \left[\left(\frac{\partial \theta}{\partial P} \right)^2 - \left(\frac{\partial \theta_m}{\partial P} \right)^2 \right] \quad . \quad . \quad (21)$$

Hence from the law of increasing entropy :

$$\left| \frac{\partial \theta}{\partial P} \right| \geqslant \left| \frac{\partial \theta_m}{\partial P} \right|$$

Consequently an adiabatic disturbance cannot increase the absolute value of the potential temperature gradient. The total increase in kinetic energy is now given by :

$$\Delta (\overline{K} + \overline{K^*}) = \frac{R}{\overline{P}} \frac{(\delta P)^2}{12} \left(\frac{\overline{P}}{P_0} \right)^{\frac{\gamma - 1}{\gamma}} \left(\frac{\partial \theta}{\partial P} - \frac{\partial \theta_m}{\partial P} \right) \quad . \quad . \quad (22)$$

and the increase in mean-flow kinetic energy by :

$$\Delta \overline{K} = \frac{(\delta P)^2}{24} \left[\left(\frac{\partial U_m}{\partial P} \right)^2 - \left(\frac{\partial U}{\partial P} \right)^2 \right] \quad . \quad . \quad . \quad (23)$$

Eq. (23) is accurate regardless of the thickness of the layer, whereas terms of the order of $(\delta P)^4$ have been neglected in the derivation of Eq. (22). Nevertheless this equation is accurate to about one per cent if $\delta P / \overline{P}$ is less than one tenth.

The change in turbulent kinetic energy can now be determined from Eqs. (22) and (23) :

$$\overline{\Delta K^*} = \frac{(\delta P)^2}{12} \left\{ \tfrac{1}{2} \left[\left(\frac{\partial U}{\partial P} \right)^2 - \left(\frac{\partial U_m}{\partial P} \right)^2 \right] + \frac{R}{\overline{P}} \left(\frac{\overline{P}}{P_0} \right)^{\frac{\gamma - 1}{\gamma}} \left[\frac{\partial \theta}{\partial P} - \frac{\partial \theta_m}{\partial P} \right] \right\} \quad . \quad (24)$$

It will be assumed that the percentage change in velocity gradient is equal to the percentage change in potential temperature gradient, which is similar to the common assumption that the eddy viscosity is equal to the eddy conductivity. If the disturbance reduces the gradients by a fraction α then :

$$\frac{\dfrac{\partial \theta_m}{\partial P}}{\dfrac{\partial \theta}{\partial P}} = \frac{\dfrac{\partial U_m}{\partial P}}{\dfrac{\partial U}{\partial P}} = 1 - \alpha \qquad 0 < \alpha < 2$$

On this assumption Eq. (24) reduces to :

$$\overline{\Delta K^*} = \frac{\alpha}{24} (\delta U)^2 \left[2 (1 - Ri) - \alpha \right]$$

where Ri is the initial Richardson number and δU is the change in velocity between top and bottom of the layer, i.e.

$$\delta U = \delta P \frac{\partial U}{\partial P}$$

Any turbulence which is present initially will cause mixing to take place and the gradients will be decreased slightly. If $Ri \geqslant 1$ this mixing will be accompanied by a decrease in turbulent kinetic energy and will only continue until the initial supply of turbulent kinetic energy is used up. This results in a decrease in gradients given by :

$$\alpha = 1 - Ri + \left\{(1 - Ri)^2 - \frac{24\,\Delta\overline{K^*}}{(\delta U)^2}\right\}^{\frac{1}{2}}$$

and an increase in Richardson number given by :

$$Ri_m = \frac{Ri}{1 - \alpha}$$

the maximum possible change occurring when $\overline{K^*} = -\Delta\overline{K^*}$, in which case all the initial turbulent energy has been used up. On the other hand if $Ri < 1$ mixing will at first cause an increase in turbulent kinetic energy and consequently an acceleration in the rate of mixing. This will continue until the gradients have been changed sufficiently to cause the Richardson number to increase to unity. At this stage the turbulent kinetic energy will be a maximum and thereafter will be used up in decreasing the gradients still further. When the turbulent kinetic energy is reduced to its initial value then the gradients will have been changed by an amount given by :

$$\alpha = 2\,(1 - Ri)$$

and the new Richardson number will be :

$$Ri_m = \frac{Ri}{1 - \alpha} = \frac{Ri}{2\,Ri - 1}$$

However small the initial amount of turbulence, the layer is capable of mixing itself to the above extent and the change is spontaneous in the sense that it is not merely caused by absorption of turbulent energy already present but this energy acts as a trigger which initiates a change from an unstable initial state to a stable final state. If Ri equals $\frac{1}{2}$ then complete mixing can occur spontaneously. When $Ri < \frac{1}{2}$ then ideally a disturbance can occur involving partial overturning which changes the sign of the gradients. It seems more probable however that complete mixing will occur, leaving a surplus of turbulent kinetic energy. The results can be summarized as follows :

$Ri \geqslant 1$ Stable. Any mixing implies a decrease in turbulent kinetic energy.

$Ri < 1$ Partial mixing can occur spontaneously.

$Ri = \frac{1}{2}$ Complete mixing can occur spontaneously.

$Ri < \frac{1}{2}$ Complete mixing leaves a surplus of turbulent kinetic energy.

$Ri < 0$ Any disturbance leaves a surplus of turbulent kinetic energy.

5. Adiabatic disturbances

The disturbance most commonly considered in the literature is overturning. This is a reversible process consisting merely of the interchange of layers without mixing; the final temperature is readily determined, either analytically or with the aid of a tephigram, since each layer conserves its potential temperature. It is not typical of the mixing processes which occur in the atmosphere and will not be discussed here in detail, though the energy changes involved in overturning will be compared with those involved in complete mixing. Complete mixing is an irreversible process and it is not possible, except in special cases, to follow the motion of individual elements on the tephigram and so determine the energy

changes. After complete mixing the final state is characterized by constant potential temperature. The change in kinetic energy caused by completely mixing a thin layer with constant initial potential temperature gradient can be found from Eq. (22) by putting $\partial\theta_m/\partial P$ equal to zero. The result is :

$$\begin{aligned}\Delta E &= \frac{\delta P}{g}\,\Delta\,(\overline{K} + \overline{K^*}) \\ &= \frac{(\delta P)^3\,R}{12\,g\,\overline{P}}\left(\frac{\overline{P}}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\frac{\partial\theta}{\partial P} \\ &= -\frac{(\delta P)^3\,T}{12\,\Gamma^2\,\overline{P}^2}\left(\frac{\gamma-1}{\gamma}\right)^2\left(\Gamma + \frac{\partial T}{\partial z}\right) \quad . \qquad . \qquad . \qquad (25)\end{aligned}$$

where Γ is the dry adiabatic lapse rate g/C_p. The neglected terms are of the order of $(\delta P)^5\,T/(\overline{P}^4\,\Gamma^2)$. If $\overline{P}$ is 1,000 mb and δP is 100 mb then the error involved in using the approximate form, Eq. (25), is only about 1 per cent. To a first approximation therefore the energy change depends on the cube of the thickness of the layer, as one would expect intuitively.

The kinetic energy change caused by the complete mixing of a layer, not necessarily thin and of arbitrary temperature distribution, can be found in the following way. Firstly conservation of potential heat, Eq. (4), gives the final potential temperature :

$$\theta_m = \frac{1}{\delta P}\int_{P_2}^{P_1}\theta\,dP = \bar{\theta}$$

Hence $$T_m = \bar{\theta}\left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}}$$

Therefore $$\Delta E = \frac{C_p}{g}\int_{P_2}^{P_1}(T - T_m)\,dP = \frac{C_p}{g}\int_{P_2}^{P_1}\left[T\,(P) - \bar{\theta}\left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]dP \quad . \qquad (26)$$

Given the initial temperature distribution this equation enables the kinetic energy change caused by complete mixing to be calculated in the general case.

Exact analytic expressions can be determined in certain cases when the temperature is given as an analytic function of the pressure. For example if it is assumed that $\partial\theta/\partial P$ is constant then :

$$\theta\,(P) = \bar{\theta} + P'\frac{\partial\theta}{\partial P}$$

$$T\,(P) = \left(\bar{\theta} + P'\frac{\partial\theta}{\partial P}\right)\left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}}$$

Therefore $$\Delta T = -\,P'\frac{\partial\theta}{\partial P}\left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}}$$

and $$\Delta E = \frac{C_p}{g}\frac{\partial\theta}{\partial P}\int_{P_2}^{P_1}P'\left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}}dP$$

$$= \frac{C_p}{g}\frac{\partial\theta}{\partial P}P_1^{\,2}\left(\frac{P_1}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\left\{\left(\frac{\gamma}{3\gamma-1}\right)\left(1-y^{\frac{3\gamma-1}{\gamma}}\right)-\left(\frac{\gamma}{2\gamma-1}\right)\left(\frac{1+y}{2}\right)\left(1-y^{\frac{2\gamma-1}{\gamma}}\right)\right\} \qquad (27)$$

where y equals P_2/P_1. The expression in the curly brackets is a function of y only, $f_1\,(y)$ say. With the aid of a graph of this function the energy necessary to mix a layer completely can easily be determined for any set of values of P_1, P_2 and $\partial\theta/\partial P$ (see Fig. 1.)

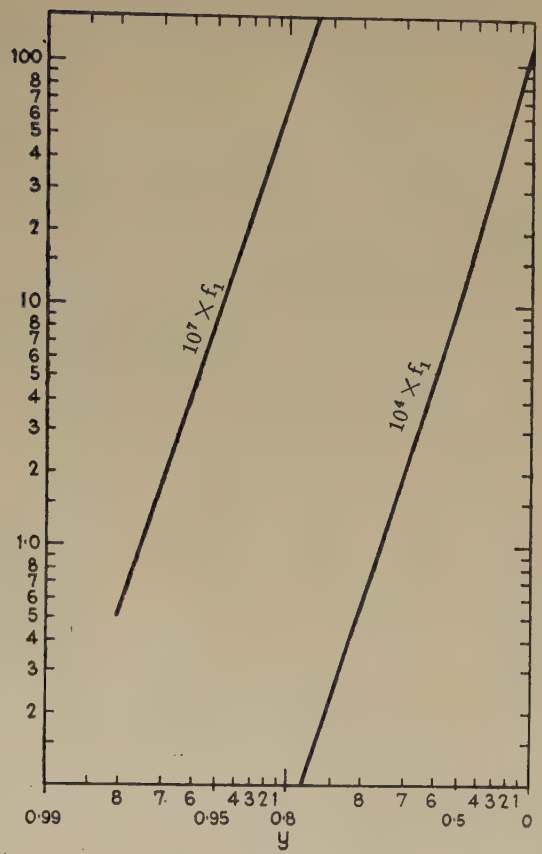

Figure 1. Graph of the function f_1.

When θ' is small compared with $\bar{\theta}$, the increase in entropy caused by complete mixing is readily determined from Eq. (20) by putting θ'_m equal to zero. It can also be seen from this equation that complete mixing gives the maximum possible increase in entropy. That this property is true regardless of the initial magnitude of θ' is easily shown by the following argument. Suppose some unspecified disturbance causes an increase in entropy ΔS. If the layer is now mixed completely there will be a further increase in entropy, ϵ say. Had the layer been completely mixed in the first place there would have been an increase in entropy $\Delta S_1 = \Delta S + \epsilon$. Hence $\Delta S_1 \geqslant \Delta S$ since ϵ cannot be negative.

When a layer is overturned the mass originally at the level P moves to the level $P_1 + P_2 - P$ hence :

$$\theta_m(P) = \theta(P_1 + P_2 - P)$$

and

$$T_m(P) = \theta(P_1 + P_2 - P)\left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}}$$

Therefore

$$\Delta T = [\theta(P_1 + P_2 - P) - \theta(P)]\left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}}$$

and

$$\Delta E = \frac{C_p}{g}\int_{P_2}^{P_1}[\theta(P) - \theta(P_1 + P_2 - P)]\left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}} dP \quad . \quad . \quad (28)$$

If it is supposed that $\partial T/\partial z$ is constant in the initial state then this integral cannot be

evaluated in terms of simple functions (Brunt 1939). On the other hand if $\partial\theta/\partial P$ is constant initially then :

$$\Delta T = -2P' \frac{\partial\theta}{\partial P}\left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}}$$

so that the temperature change caused by overturning is twice that caused by complete mixing. It follows immediately that the kinetic energy change on overturning a layer in which the potential temperature is a linear function of pressure is also exactly twice the change on completely mixing it.

An approximately dry adiabatic temperature distribution can sometimes be produced by moving half the mass at the level P to $P_1 + P_2 - P$, leaving the other half at P. This is repeated for all P in the layer and the halves are mixed at their destination levels. This system of mixing was given by Normand (1946). It will produce an exactly dry adiabatic temperature distribution only in certain cases, e.g. if $\partial\theta/\partial P$ is initially constant. On the other hand if $-\partial T/\partial z = \beta$, a constant, then :

$$\theta_m(P) = \frac{\theta(P_1)}{2}\left\{\left(\frac{P}{P_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{\beta}{\Gamma}-1\right)} + \left(\frac{P_1+P_2-P}{P_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{\beta}{\Gamma}-1\right)}\right\}$$

In this case $\theta_m(P)$ is clearly not constant. If the layer is initially stable then the lower half is unstable after mixing.

6. The mixing of a layer with the addition of heat

Suppose a quantity of heat Q is added at pressure P_3 within the layer P_2 to P_1; the work necessary to redistribute this heat in such a manner that the potential temperature gradient is unchanged will now be calculated. The addition of heat Q is equivalent to the addition of a quantity of potential heat $Q\left(\frac{P_0}{P_3}\right)^{\frac{\gamma-1}{\gamma}}$. This potential heat must be redistributed uniformly throughout the layer if the potential temperature gradient is to remain the same. The increase in potential temperature at every point is therefore $\frac{g}{C_p}\frac{Q}{\delta P}\left(\frac{P_0}{P_3}\right)^{\frac{\gamma-1}{\gamma}}$ and consequently the transfer of potential heat is given by :

$$F = \frac{Q}{\delta P}\left(\frac{P_0}{P_3}\right)^{\frac{\gamma-1}{\gamma}}(P - P_1) \qquad P_3 < P < P_1$$

and

$$F = \frac{Q}{\delta P}\left(\frac{P_0}{P_3}\right)^{\frac{\gamma-1}{\gamma}}(P - P_2) \qquad P_2 < P < P_3$$

The kinetic energy change can now be determined from Eq. (13) :

$$\Delta E = \left(\frac{\gamma-1}{\gamma}\right)\int_{P_2}^{P_1}\frac{F}{P}\left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}} dP.$$

$$= Q\left\{1 - \left(\frac{\gamma}{2\gamma-1}\right)\left(\frac{P_1^{\frac{2\gamma-1}{\gamma}} - P_2^{\frac{2\gamma-1}{\gamma}}}{\delta P\, P_3^{\frac{\gamma-1}{\gamma}}}\right)\right\} \qquad . \qquad . \qquad (29)$$

Hence

$$\Delta E = Q\left\{1 - \left(\frac{\gamma}{2\gamma-1}\right)\frac{\left(1 - y^{\frac{2\gamma-1}{\gamma}}\right)}{(1-y)\,x^{\frac{\gamma-1}{\gamma}}}\right\}$$

The function, f_2 say, in the curly brackets is a function of x and y only, where y equals P_2/P_1 and x equals P_3/P_1. This function has been tabulated and graphed (see Table 1 and Fig. 2). The values of x for which f_2 vanishes have been specifically calculated.

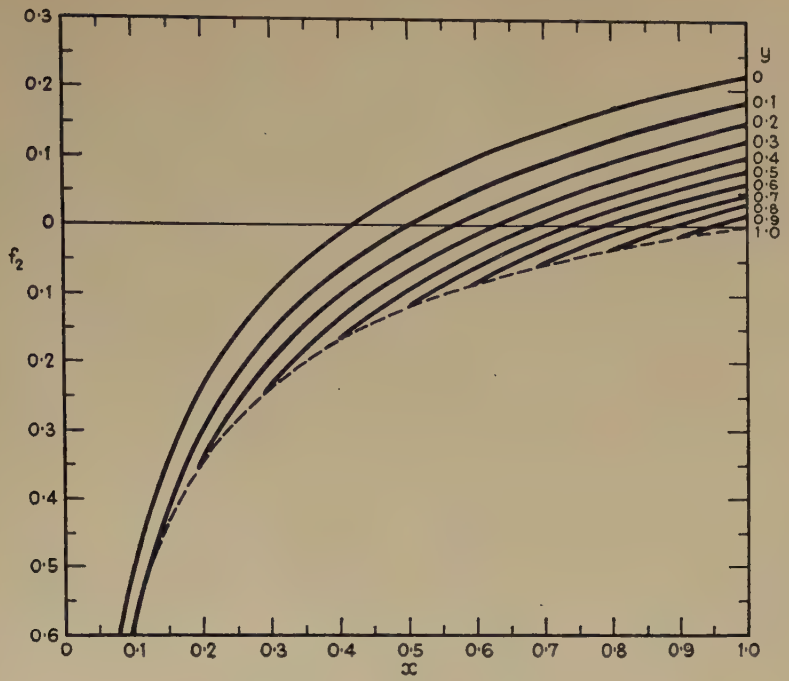

Figure 2. Graph of the function f_2.

These values give the ' neutral ' pressure within the layer, no kinetic energy change occurring when the layer is mixed if the heat is added at the neutral pressure. If heat is added above the neutral pressure or removed from below the neutral pressure then a supply of kinetic energy is necessary to mix the layer; if heat is added below the neutral pressure or removed from above the neutral pressure then kinetic energy is released on mixing the layer.

When the disturbed layer comprises the bulk of the atmosphere above the level P_1, then P_2 may be neglected, and Eq. (29) reduces to :

$$\Delta E = Q\left\{1 - \left(\frac{\gamma}{2\gamma - 1}\right)\left(\frac{P_1}{P_3}\right)^{\frac{\gamma-1}{\gamma}}\right\}$$

If the heat is added at the base of the layer, i.e. if P_3 equals P_1, then the kinetic energy released is $Q\left(\frac{\gamma - 1}{2\gamma - 1}\right)$. This means that 22 per cent of the heat supplied at the base would be converted to kinetic energy if the layer were kept mixed in such a manner that the potential temperature gradient remained constant. The ' neutral ' pressure in this case is given by :

$$P_3 = P_1\left(\frac{\gamma}{2\gamma - 1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= 415 \text{ mb if } P_1 = 1{,}000 \text{ mb}$$

Furthermore the addition of heat at great heights will cause very great stability which a much larger degree of heating at the base will not dispel.

Discussion has been limited to the case of addition of heat to one level only. The methods of this section however can easily be applied to situations where the heat is added in any way.

TABLE 1. COMPUTATION OF KINETIC ENERGY CHANGE (ΔE) PRODUCED BY ADDITION OF HEAT (Q) AT A LEVEL (P_3) IN A LAYER BOUNDED BY LEVELS P_1 AND P_2

$$f_2 = \frac{\Delta E}{Q} = 1 - \left(\frac{2\gamma-1}{\gamma}\right) \frac{\left(1 - y^{\frac{2\gamma-1}{\gamma}}\right)}{(1-y)\, x^{\frac{\gamma-1}{\gamma}}} \qquad y = \frac{P_2}{P_1} \qquad x = \frac{P_3}{P_1}$$

y \ x	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0	values of x when $f_2 = 0$
0	∞	− 0·5016	− 0·2318	− 0·0971	− 0·0105	+ 0·0519	0·1000	0·1388	0·1710	0·1984	0·2222	0·4150
0·1		− 0·5820	− 0·2978	− 0·1558	− 0·0647	+ 0·0011	0·0518	0·0927	0·1266	0·1555	0·1806	0·4980
0·2			− 0·3453	− 0·1982	− 0·1038	− 0·0355	+ 0·0171	0·0594	0·0946	0·1246	0·1506	0·5650
0·3				− 0·2339	− 0·1366	− 0·0664	− 0·0122	+ 0·0316	0·0676	0·0985	0·1252	0·6262
0·4					− 0·1657	− 0·0937	− 0·0382	+ 0·0065	0·0437	0·0754	0·1028	0·6842
0·5						− 0·1185	− 0·0617	− 0·0160	+ 0·0221	0·0544	0·0825	0·7400
0·6							− 0·0833	− 0·0366	+ 0·0022	0·0352	0·0638	0·7940
0·7								− 0·0559	− 0·0164	+ 0·0173	0·0464	0·8467
0·8									− 0·0339	+ 0·0004	0·0300	0·8988
0·9										− 0·0154	+ 0·0147	0·9495
1·0											0·0000	1·0000

7. The frost fan

The results of the preceding two sections can be used to solve the frost-fan problems mentioned in the introduction. Firstly the energy required to destroy an established inversion at the earth's surface can be determined from Eq. (25). It indicates that approximately 10^4 ergs cm^{-2} are required to mix the lowest thirty metres of the atmosphere with an inversion of 5°C. Alternatively 10^4 ergs cm^{-2} are released when the first thirty metres of the atmosphere are mixed if there is a lapse of 5°C. This energy will appear as turbulent kinetic energy and 10^4 ergs cm^{-2} is equivalent in this case to about 3 ergs cm^{-3} which will give typical turbulent velocities of the order of 70 cm sec^{-1}.

The solution to the second frost-fan problem i.e. the energy required to maintain a completely mixed state when a layer is continuously cooled from below, can be found from Eq. (29). When applied to a thin layer this equation reduces to :

$$\Delta E = Q\left(\frac{\gamma - 1}{\gamma}\right)\left(\frac{P_3}{\bar{P}} - 1\right).$$

$$= Q\left(\frac{\gamma - 1}{\gamma}\right)\frac{\delta P}{2\bar{P}} \quad \text{if} \quad P_3 = P_1.$$

In the case of the frost fan Q is negative and kinetic energy is absorbed in maintaining the mixed state. The rate at which kinetic energy is being absorbed will give the minimum power required for a frost fan, which Squires (1948) calls the radiation load on the fan. If Q/t is about -10^5 ergs cm^{-2} sec^{-1} where t is the time taken to add Q ergs cm^{-2} and if the mixing extends through a depth of 30 metres then :

$$-\frac{\Delta E}{t} = 45 \text{ ergs cm}^{-2} \text{ sec}^{-1}$$

$$= \tfrac{1}{4} \text{ HP/acre}$$

This value is very much smaller than the actual power used by frost fans, which is of the order of 12 HP/acre. Apparently most of the kinetic energy generated by the fan is either used in peripheral mixing which does not extend to ground or dissipated by viscous forces resulting in the inefficient direct heating of the air. A large-diameter slow-speed fan is inefficient from the former point of view and a small-diameter high-speed fan is inefficient from the latter, the ideal being a compromise between the two extremes. If 6 HP/acre is dissipated by viscous forces this is equivalent to about 300 ergs gm^{-1} sec^{-1} and is of the same order as the viscous dissipation which occurs naturally in the first 30 m of the atmosphere when the wind speed is about 5 m/sec (Taylor 1952). The rate of gain of heat resulting from this dissipation is only about one-hundredth of the rate of loss of heat caused by cooling at the ground and is therefore insignificant from the point of view of frost prevention.

Acknowledgment

I am indebted to Mr. W. C. Swinbank for assistance and advice during the preparation of this paper.

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The influence of the 9·6 micron ozone band on the atmospheric infra-red cooling rate

By G. N. PLASS

*The Johns Hopkins University, Baltimore, U.S.A.**

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SUMMARY

The upward and downward radiation flux and the heating and cooling rate in the atmosphere have been determined for the 9·6 μ band of ozone from the absorption measurements of Summerfield. Three different ozone distribution curves as well as three different temperature-height curves have been used for these calculations. The radiations that interact with the 9·6 μ ozone band always act to cool the atmosphere from the earth's surface to a height of several kilometres and to heat the region from there to approximately 20 km. The heating or cooling rate is never larger than a few tenths of a degree Celsius per day in this region. On the other hand, from 35 to 60 km the usual order of magnitude of the cooling is 2 to 3°C/day, but the cooling rate can be considerably larger if the upper layers have a higher temperature than normal. An increase in the ozone amount at the lower altitudes may cause the surface temperature of the earth to rise several degrees Celsius in order to restore infra-red equilibrium, if no other factors change that produce an effect upon the heat balance. The pressure broadening and overlapping of the spectral lines have been taken into account in these calculations together with all other physical phenomena that are known to be of importance to the problem. An estimate is given for the accuracy of the results.

1. INTRODUCTION

This study is concerned only with the influence of the 9·6 μ ozone band on the infra-red radiation flux and the heating and cooling rates in the atmosphere. The upward and downward flux and the cooling rates are calculated here for three different ozone distributions and for three different temperature-height curves. The results are obtained from the absorption measurements of Summerfield (1941). Such effects as the pressure broadening and overlap of the spectral lines are taken into account. The atmosphere is not divided into homogeneous layers for the purpose of the calculation, as this procedure may introduce large errors in the determination of cooling rates. At each stage a careful estimate is made of the accuracy of the calculation.

The many aspects of the ozone problem have been discussed in the excellent survey articles by Craig (1950, 1951); these contain references to earlier work. Special mention should be made of the long study of ozone cooling rates by Gowan, of which the most recent was published in 1947. Recent articles by Wexler (1950), Craig (1951), Johnson (1953, 1954) and Pressman (1955) have made interesting contributions to the ozone problem and have summarized more recent work.

2. METHOD OF CALCULATION OF ATMOSPHERIC TRANSMISSION FROM LABORATORY DATA

Laboratory measurements of the transmission of infra-red bands as a function of pressure and path length can be used for the calculation of the radiation flux in the atmosphere. The method used in this article has been described by Plass (1952a). From various types of graphs of the laboratory absorption measurements, it is possible to find the regions of validity of several different approximations for the transmission function and to estimate the accuracy of the approximation in each case. It is then possible to calculate the infra-red radiation flux and cooling rates in the atmosphere directly from the laboratory data.

* This paper was completed while the author was on leave of absence at Michigan State University. At present with Lockheed Aircraft Corporation, Van Nuys, California.

When these calculations were first made in 1952 the most accurate absorption measurements for the 9·6 μ band of ozone were those of Summerfield (1941). His results have been analysed by a different procedure by Elsasser and King (1953). More recent ozone measurements have recently been discussed by Goody and Walshaw (1954) and by Walshaw and Goody (1954). However, the calculations were completed using Summerfield's results, as they appear to be sufficiently accurate and complete for this purpose.

All Summerfield's measured points at pressures less than atmospheric are shown in Fig. 1, where the logarithm of the fractional absorption, A, is plotted against the logarithm of the product of the pressure, p, and the path length, w, used in the laboratory. It is seen that all the measured points fall on a single universal curve within the experimental error of about 5 per cent. Thus the experimental results show that the ozone absorption can be represented by this single universal curve for the range of pressure and path length used in the experiment (this includes a large portion of the relevant range in the atmosphere). The physical meaning of this result can be shown to be (Plass, 1952a) that most of the absorption is caused by the wings of the strong lines for this range of pressure and path length. Less than 5 per cent of the absorption can come from the centres of weak lines; there would be noticeable departures from the universal curve if this were not true.

From this universal curve, representing the laboratory measurements, Plass (1952a) has shown that the transmission along the variable path conditions found in the atmosphere can be obtained by replacing the variable pw by $g^{-1} \sec\theta \int_{p_0}^{p_1} c\, p\, dp$, where g is the

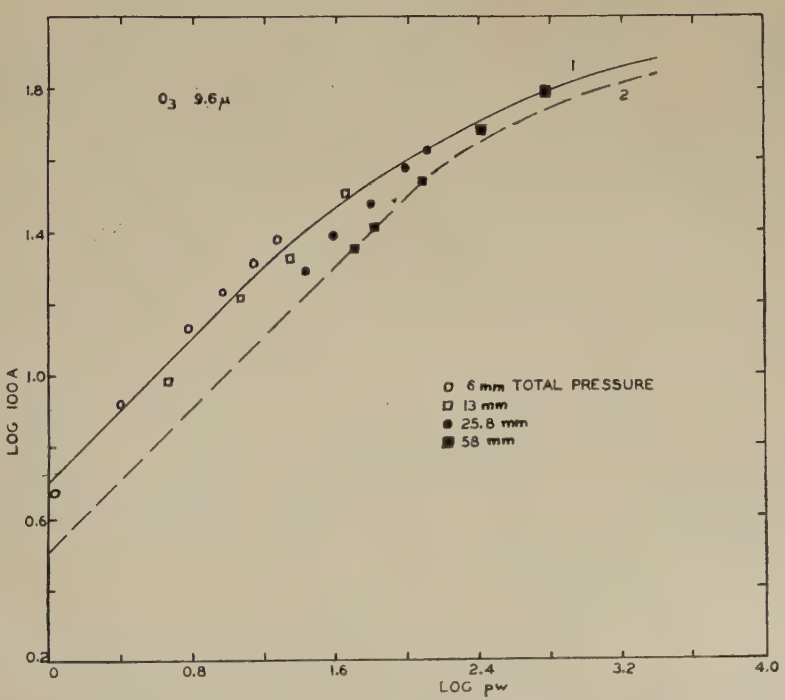

Figure 1. The fractional absorption of the 9·6 μ ozone band as measured by Summerfield (1941). A is the fractional absorption measured in the laboratory; p is the pressure in mm; w is the path length through the ozone in mm at STP. Absorption curve 1 was taken as giving the best fit of the experimental data. Absorption curve 2 was drawn to lie outside the experimental limits of error; its use provided a check of the sensitivity of the results to changes in the absorption curve.

acceleration of gravity, θ is the angle that the beam makes with the vertical, c is the ratio of the density of the radiating gas, ρ_r, to the total density, ρ, and p_0 and p_1 are the pressures at the two levels between which one wishes to calculate the transmission.

This substitution of variables gives a result for the transmission along the variable atmospheric path that is as accurate as the original laboratory measurements. This substitution is valid regardless of how irregularly the line spacing, intensity, or half-width may vary throughout the band, and of whether or not the lines overlap, provided that the range of pressure and path length is in the experimentally determined region of validity of the universal curve. This result is also valid for any line shape which has an absorption coefficient proportional to the pressure in the wings of the line, as is true for the Lorentz line shape or any other collision-broadened line shape that has been proposed.

It is also assumed for the ozone calculation that the measured absorption for the entire band is independent of the temperature. Such an assumption of temperature independence would not be justified, for example, for the 15 μ band of carbon dioxide, since this must be divided into several regions about 1 μ wide for a calculation of this type. In order to use this same method for carbon dioxide, a procedure has been developed for correcting the universal curve by using the Boltzmann factor and an effective temperature between the levels of the atmosphere in question. This method will be discussed in detail when we present the results of our carbon dioxide calculations. When this method is applied to ozone, we find that the temperature correction does not change the absorption for the entire band by more than the experimental error in Summerfield's measurements for the range of temperatures in the atmosphere. Since this result agrees with theoretical predictions for the temperature dependence of the absorption from an entire band, we do not present further details here. Laboratory measurements of the absorption of atmospheric gases should ideally be taken over the range of temperatures that occurs in the atmosphere. This would eliminate any uncertainties that may be connected with the theoretical estimates for this correction. However, to our knowledge, no suitable measurements have yet been made at temperatures below room temperature.

When the logarithm of the absorption is plotted against the logarithm of pw, as in Fig. 1, the slope of the curve is zero for virtually complete absorption. As the absorption decreases, the slope increases to the value one-half when the square-root absorption region is reached. In between these regions the slope gives a convenient measure of the degree to which the spectral lines overlap. If still smaller values of the absorption were measured, the slope would increase to unity in the linear absorption region. In the latter region, there would be no single absorption curve valid over a wide range of the variables p and w as in Fig. 1. Here another procedure described by Plass (1952a) can be used. Since the smaller absorptions shown in Fig. 1 are still above the linear region, it follows that the absorption curve must have a slope of one-half in this region.

The upward and downward flux of infra-red radiation, $I^{\uparrow}$ and $I^{\downarrow}$ respectively, can be obtained from the transmission function, τ, from the well-known solutions of Kirchhoff's equations

$$I^{\uparrow}(u) = I_b(u) + \int_u^{u_1} \tau(u, u') \frac{d\, I_b(u')}{du'}\, du' \quad . \quad . \quad . \quad . \quad (1)$$

$$I^{\downarrow}(u) = I_b(u) - I_b(u_0)\, \tau(u_0, u) - \int_{u_0}^{u} \tau(u', u) \frac{d\, I_b(u')}{du'}\, du' \quad . \quad . \quad (2)$$

where

$$\tau(u, u') = \exp\left\{ - \sec\theta \int_u^{u_1} k(u)\, du \right\} \quad . \quad . \quad . \quad (3)$$

$$du = - c\, \rho\, dz = - \rho_r\, dz,$$

$I_b(u)$ is the black-body intensity at the level u, θ is the angle the radiation makes with the vertical, u_0 and u_1 are the upper and lower boundaries in the atmosphere ($u_0 < u_1$), and k is the absorption coefficient. It is assumed that the temperature of the atmosphere at the lower boundary ($u = u_1$) is the same as the temperature of the surface and that the surface emits as a black body. This is the case if the lower boundary is at the surface of the earth or at the upper surface of a cloud. It is also assumed that no appreciable flux is incident on the atmosphere from outside the earth at 9·6 μ.

The time rate of change of the temperature due to the flux of infra-red radiation in the interval $\Delta\nu$ is given by :

$$\frac{\partial T}{\partial t} = \frac{c\,\Delta\nu}{c_p}\frac{d}{du}(I^{\uparrow} - I^{\downarrow}) \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where c_p is the specific heat at constant pressure of the air. Thus the cooling or heating rate can be obtained from the derivative of the difference of the upward and downward flux.

Although the upward and downward flux can probably be calculated to a reasonable accuracy by the division of the atmosphere into a number of homogeneous layers, numerical examples convinced us that this procedure can introduce large errors in the derivatives that are needed to obtain the heating and cooling rates from Eq. (4). Further, such calculations showed that large additional errors can be introduced in the final results if intervals larger than 1 km are taken for the numerical evaluation of Eqs. (1), (2) and (4).

The actual procedure adopted for the evaluation of Eqs. (1) and (2) was to perform the indicated integrations numerically by the choice of intervals of 1 km or less, depending on the rate at which the function varied. In the calculations, all atmospheric quantities were taken to have their actual continuous variation with height and were not averaged over layers. The transmission functions were obtained from the data of Summerfield and corrected where necessary by the methods described below. As the calculation of each curve shown in the following sections required well over 10,000 arithmetic operations, the problem was finally set up for an IBM electronic calculator.

Although the straightforward procedure for the calculation of the flux and cooling rates outlined above could be used for most of the intervals, there were certain, usually small, corrections that had to be made between certain levels in the atmosphere. At the highest elevations for which these calculations were made (up to 70 km), it was necessary to consider the importance of Doppler broadening to the line shape in addition to the Lorentz broadening. This correction can be made without difficulty from the equations given by Plass and Fivel (1953). A small correction was made to the transmission data to account for the fact that Summerfield did not include the lines at the extreme edges of the ozone band in his measurements. This correction was made by the method outlined by Elsasser and King (1953).

When the path length is quite short, particularly at the higher altitudes, the equivalent product of the pressure and path length is so small that a large extrapolation has to be made beyond the measured values in Fig. 1. For a sufficiently short path length, the transmission is determined by the total line intensity of the band (Plass 1952a), which can be obtained in the usual manner (Elsasser and King 1953). Because of the lack of experimental evidence in this case for the region of validity of the linear approximation, this was estimated from the equations given by Plass (1954). Although there is a larger uncertainty associated with the points that had to be corrected in this manner, a careful examination of the results of a number of trial runs has shown that this correction influences the final results only at the highest altitude. This uncertainty has been taken into account in estimating the accuracy of these results. Thus, the universal curve is not used for short path lengths.

The measurements of Summerfield show that the absorption cannot be represented by a single universal curve when the pressure is near one atmosphere and for a moderate path length. Since the cooling rate for ozone does not need to be known accurately for the first few kilometres above the earth's surface, no correction was made in this calculation for this departure from the universal curve at pressures near one atmosphere.

In order to compute the total diffuse infra-red radiation for the band, it is necessary to integrate not only over the frequency, but over the solid angle of the hemisphere. This latter integration has often been taken into account approximately by multiplying the path length by some constant factor near 1·67. Because this factor does vary between one and two, as the transmission in an interval changes from one to zero, certain errors can be introduced in calculations of this type by using such a constant factor (Plass 1952b). However, even with the aid of an electronic computor to perform the final integrations, it appeared to be prohibitively complicated to calculate the integrals over the hemisphere for each of the curves given here. It was therefore decided to multiply the path lengths by a numerical factor to convert into radiation over a hemisphere. In order to improve the accuracy of the usual procedure of using a constant numerical factor, a numerical factor was chosen that depended on the transmission between the levels in question.

The relationship between these two quantities was calculated from the results of Plass (1952b). This procedure was checked by actually performing the numerical integration over the hemisphere in several cases. These calculations, together with the results of Plass (1952b), show that the procedure adopted here of using a numerical factor that depends on the transmission in the interval, cannot introduce an error as large as one per cent in the transmission function.

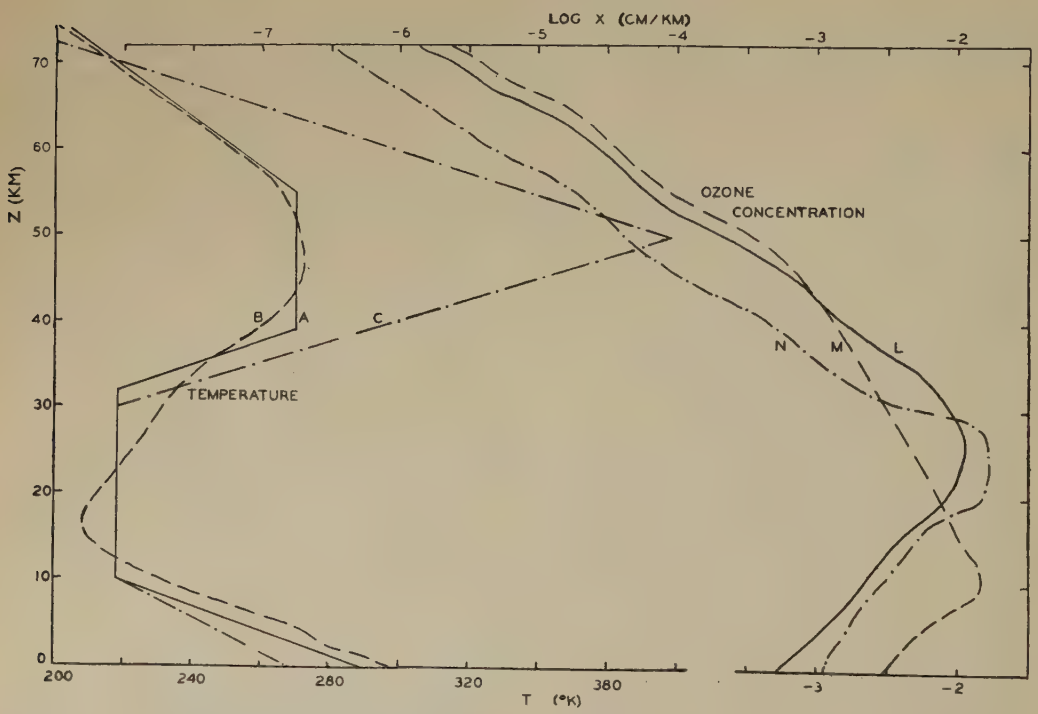

Figure 2. Assumed variation with height of the ozone concentration (curves L, M, N) and of the temperature (curves A, B, C). T is the absolute temperature; x is the ozone amount in cm/km at STP; z is the height in kilometres.

3. Assumed ozone and temperature distributions

Various combinations of the three different ozone distribution curves and of the three temperature-height curves shown in Fig. 2 were used in the six different calculations reported here. The ozone distribution curve L agrees with the NRL rocket measurements of 14 June 1949 above 20 km (Craig 1950, Johnson 1954). This is believed to represent the average ozone concentration at middle latitudes with reasonable accuracy. The curve was extended to lower altitudes from the average of other measurements.

The APL rocket measurements of 25 Jan. 1951 were used to determine the ozone distribution curve M above 20 km. This curve represents larger ozone concentrations above 45 km than curve L. At lower altitudes curve M gives the smallest ozone amounts that have ever been measured at these altitudes. For the total ozone amount to be reasonable, the maximum of this curve must be at a very low altitude. The curve has been continued below 20 km in such a manner. A careful analysis shows that this distribution probably existed at the time of the experiment, possibly owing to convergence and subsidence at the lower altitudes. It would be difficult to explain this result as due to instrumental failure. In any case, we have assumed the ozone distribution curve M in order to study the influence on the infra-red flux and cooling rates of a larger ozone concentration at high altitudes and of a maximum in the ozone distribution at a relatively low altitude.

The NRL measurements of 10 Oct. 1946 are the basis for ozone distribution curve N. This curve has been extended to high altitudes and to the surface of the earth to give a reasonable total amount of ozone according to other measurements. This measurement showed a very much smaller ozone amount above 30 km than is usually found. Further-

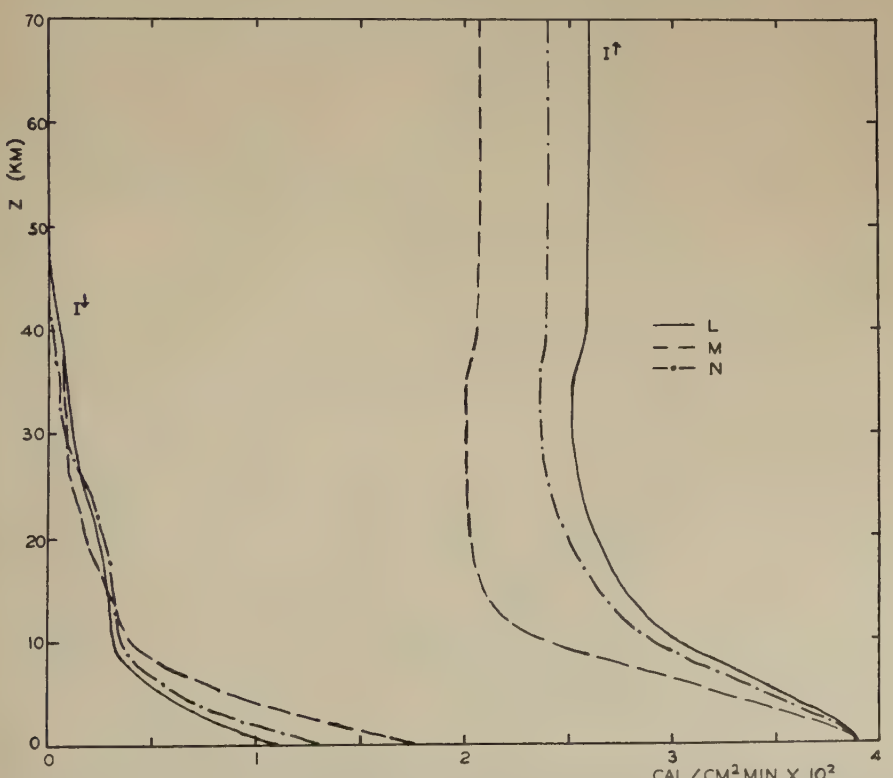

Figure 3. The upward and downward radiation flux ($I^{\uparrow}$ and $I^{\downarrow}$ respectively) in the region of the 9·6 μ ozone band for ozone-distribution curves L, M, N, temperature curve A, and transmission curve 1.

more, the maximum around 23 km is relatively narrow and shows one of the highest ozone amounts (0·17 cm/km) that has been measured. We have assumed the ozone distribution curve N in order to study the effect of greatly reducing the ozone amount at high altitudes and of having a narrow, intense maximum in the distribution. The total ozone amounts for curves L, M and N are 0·213, 0·267 and 0·254 cm respectively at standard temperature and pressure. The curves in Fig. 2 have been plotted against the logarithm of the concentration as abscissa and not against the concentration directly as is customary.

Temperature curve A, the first of the three temperature-height curves used in this study, was chosen to give a reasonably accurate representation of the variation of the average temperature with height at middle latitudes. The calculations are considerably simplified if the temperature-height curve is composed of a number of straight-line segments, since the number of arithmetical operations required to obtain a given accuracy is then reduced. Curve A was taken as our standard temperature curve and curves B and C were introduced in order to study the effect of temperature variations from those given by curve A.

Temperature curve B agrees with the values adopted by the Rocket Panel (1952) as the best average temperatures that can be given at the present time. We repeated the calculations using curve B in order to study the effects of the discontinuities of slope in curve A on the cooling rate.

Temperature curve C was arbitrarily drawn to study the effect of a sudden heating in the upper atmosphere (to a maximum of 378°K at 50 km) and of somewhat lower temperatures near the surface of the earth than are given by curve A. All of the densities used in this calculation are those given by the Rocket Panel (1952) corrected where necessary for our assumed variation of the temperature.

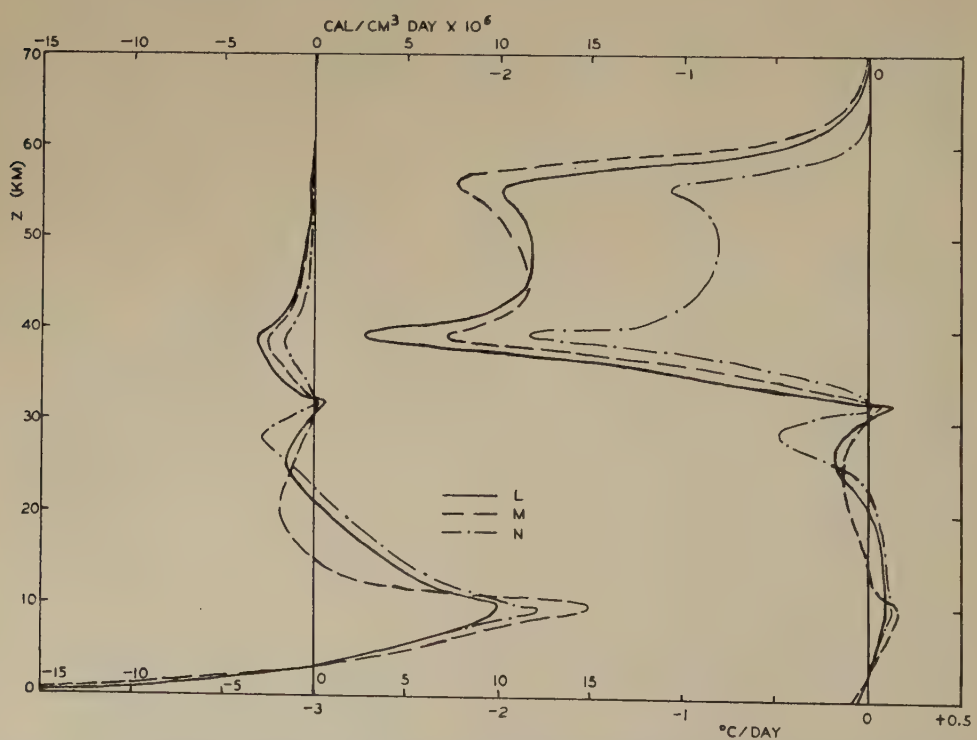

Figure 4. The heating or cooling rates for ozone-distribution curves L, M, N, temperature curve A, and transmission curve 1. The three curves at the left-hand side of the figure give the energy absorbed in units of cal cm^{-3} day^{-1}. The three curves at the right-hand side of the figure give the heating rate in °C/day.

4. Influence of ozone concentration on infra-red flux and cooling

In order to determine the influence of changes in the ozone concentration alone, all calculations reported in this section were made with the temperature curve A. The upward and downward infra-red flux in the region of the 9·6 μ ozone band is shown in Fig. 3 for ozone distribution curves L, M and N. All values of the flux have been reduced to an interval 1·07 μ wide centred at 9·6 μ. Curve M has the highest ozone concentration at low altitudes and therefore the upward flux is more nearly able to follow the decrease in the black-body intensity with height up to 10 km. For this reason the values of the upward flux are less for curve M at all heights than for N and L. The transmission is so near unity above 30 km that the temperature rise in this region causes only a small increase in the upward flux. The variations in the downward flux shown in Fig. 3 can be explained by similar considerations.

It is not apparent from Fig. 3 that the slopes of these curves can be determined with sufficient accuracy to calculate the heating and cooling rates from Eq. (4). However, the original results are sufficiently accurate for the numerical calculation of the derivative. This procedure was checked in several different ways and is discussed further in a later section.

The rate of heating or cooling due to the absorption and emission of infra-red radiation by the ozone is plotted in Fig. 4. Any gas must cool the atmosphere by the action of infra-red radiation in the layer next to the earth's surface, if it is assumed that the atmospheric boundary layer is at the same temperature as the earth's surface. This layer of cooling is found to exist for each ozone distribution curve in Fig. 4. If the mixing ratio is constant, or decreases with height, (as is usually the case for carbon dioxide and water vapour), the action of the infra-red radiation can only cool the atmosphere at all levels. However,

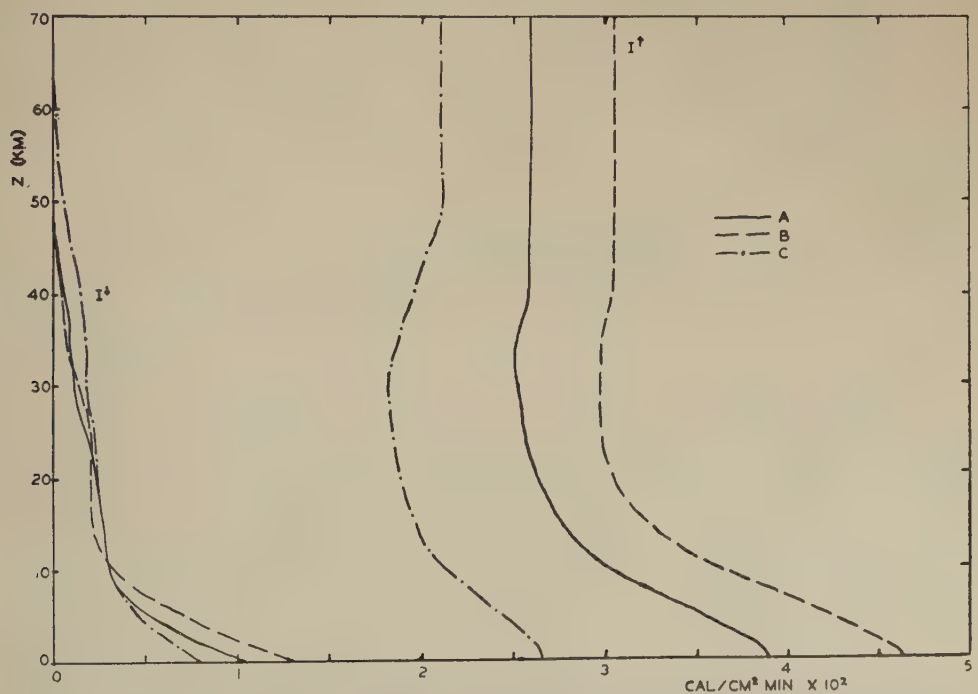

Figure 5. The upward and downward radiation flux ($I^{\uparrow}$ and $I^{\downarrow}$ respectively) in the region of the 9·6 μ ozone band for ozone-distribution curve L, temperature curves A, B, C, and transmission curve 1.

if the mixing ratio increases with height (as is the case with ozone up to heights of 10 to 30 km), then the infra-red radiation can cause a net heating of the atmosphere over a range of heights. Fig. 4 shows that this effect is most pronounced at moderate altitudes for curve M which has the highest ozone concentration at low altitudes. The ozone distribution M causes cooling above 13 km, whereas ozone distributions L and N continue to warm the atmosphere to heights of 20 km and above. The reason for this difference is the greater altitude at which the maximum of the ozone distribution occurs for curves L and N than for M. The heating rates are quite small, never exceeding 0·2°C/day at these altitudes.

The cooling rates begin to become large around 35 km for all three curves and are appreciable up to 60 km. The maximum cooling shown for any of these curves is 2·7°C/day. The cooling rate for curve N is roughly one-half that of the other curves above 40 km, because of the very small ozone concentration assumed at these altitudes for this curve. The double maximum is caused by the discontinuities in the assumed temperature curve and is discussed in the next section.

The normal variations in ozone amount in the atmosphere at a given altitude are probably represented by the three rather different values for the ozone concentrations given by curves L, M and N at each altitude. Thus the results of Figs. 3 and 4 show the approximate variation in infra-red flux and cooling rate that might be expected to occur in the atmosphere owing to reasonable variations in the ozone amount. It appears that variations in the ozone amount can change the cooling rate by as much as a factor of two. However, if the temperature remains unchanged, the qualitative features of the cooling curve, including the positions of the maxima, do not vary appreciably with changes in the ozone concentration.

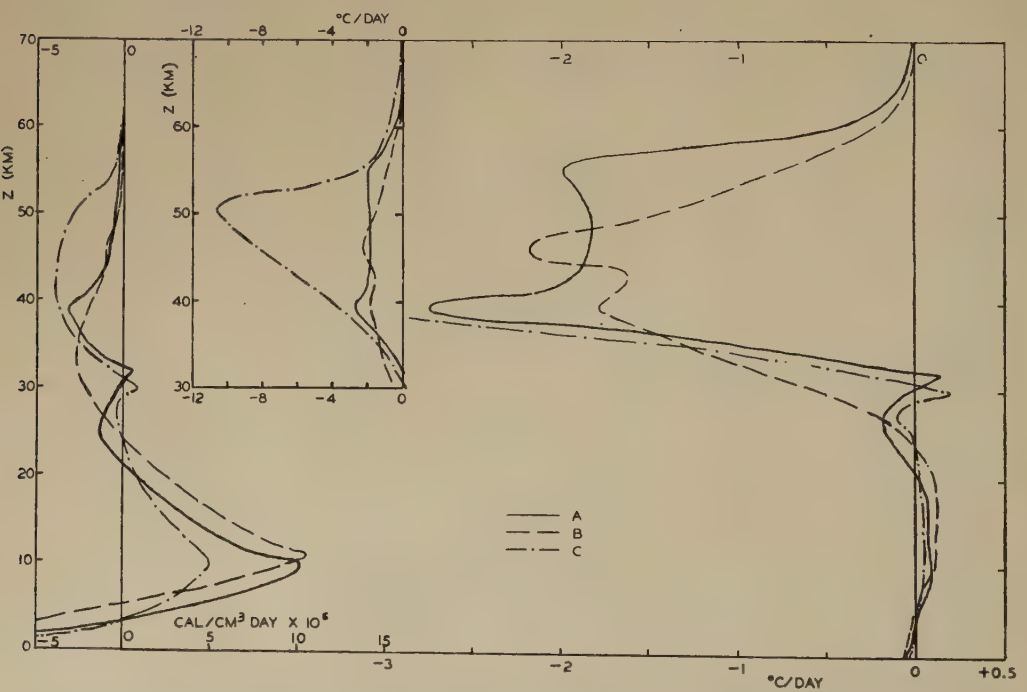

Figure 6. The heating or cooling rates for ozone-distribution curve L, temperature curves A, B, C, and transmission curve 1. The three curves at the left-hand side of the figure give the energy absorbed in units of cal cm^{-3} day^{-1}. The three curves at the right-hand side of the figure give the heating rate in °C/day. The insert in the upper part of the figure gives the heating rate in °C/day at the higher altitudes plotted on a different scale.

5. Influence of temperature distribution on infra-red flux and cooling

In order to separate the effects due to temperature variation with height from those due to changes in the ozone distribution curve, the calculations were repeated using the single ozone distribution curve L, with the three different temperature curves A, B and C (Fig. 2).

The corresponding upward and downward flux of radiation is shown in Fig. 5. Since the temperature in the atmospheric layers close to the ground is the principal factor that determines the magnitude of the upward flux for a given ozone distribution, it follows that, at all heights, the upward flux corresponding to the temperature curve C is less than that from curve A, which in turn is less than that from curve B. The assumed ground temperatures increase in the same sequence. The upward flux corresponding to curve C continues to increase at higher altitudes (up to 50 km) than for the other curves due to the very high assumed temperature at 50 km for curve C. Similarly the downward flux is considerably greater for curve C than for the other two at all altitudes above 30 km. Near the surface of the earth, the downward flux is mainly determined by the temperature of the lowest atmospheric layers. Below 7 km, the three values for the downward flux are in the same numerical sequence as the corresponding assumed temperatures.

In Fig. 6 the heating and cooling rates are shown for these three temperature curves. The region of cooling is seen to extend from the earth's surface to a few kilometres with a region of heating from there to above 20 km. The magnitude of the heating or cooling is again less than a few tenths of a degree per day in this region.

The effect introduced by use of a temperature curve composed of regions with a linear variation of temperature with height and discontinuities in the derivative (curve A), compared with a temperature distribution with a continuous derivative (curve B), is shown in Fig. 6. The points at which the derivative of the temperature with respect to height are discontinuous, appear in Fig. 6, curve A, as maxima in the cooling curve at 39 and 55 km and a narrow maximum of heating at 32 km. In order to obtain these differences, it is necessary to take quite narrow intervals for the calculation of the integrals in Eqs. (1) and (2). It is possible that our maxima would be even sharper had we used still smaller intervals in our calculation. However, it was not felt worth while to do this, since such discontinuities do not actually occur in the atmosphere. Thus a somewhat smoothed curve is closer to reality.

However, in order to check our numerical results further, we have obtained analytic solutions to similar problems by the method of Plass and Fivel (1955), where a discontinuous temperature derivative is assumed and the variation of the Lorentz line shape with pressure is taken into account. The results of these calculations confirm the conclusion that such maxima and minima are always to be expected at points of discontinuity of the derivative of the temperature distribution.

The double maximum near 40-45 km in the cooling curve for temperature distribution B, appears to be real for our assumed data. However, it may be due entirely to small variations in our assumed temperature and density curves. Otherwise curves A and B are qualitatively similar, except that curve B does not have the variations at 32, 39 and 55 km due to the discontinuities in the derivative of the temperature for curve A.

The high assumed temperatures near 50 km for curve C give such a high cooling rate above 40 km that it has been plotted on another scale as an insert to Fig. 6. There is a narrow region of heating near 30 km and a maximum cooling rate at 50 km; these are the positions of the discontinuities in temperature curve C. The cooling rate is 10·6°C/day at 50 km where the assumed temperature is 105°C compared with a maximum of 2·7°C/day for curve A at 39 km where the assumed temperature is — 3°C. Above

30 km the results of Fig. 6 agree qualitatively with the earlier calculations of Gowan (1947) (see also the discussion by Wexler 1950).

Thus if some process, such as increased ultra-violet radiation from the sun, increases the temperature of the upper layers of the atmosphere, the infra-red radiation immediately acts to cool these layers. The influence of the carbon dioxide and possibly also of the water vapour on the infra-red radiation at these levels would considerably increase the cooling rate which has been calculated here only for ozone. Thus the infra-red radiation would allow the atmosphere to return to its equilibrium temperature at a relatively rapid rate. These calculations show that the infra-red cooling rate is more sensitive to temperature variations in the atmosphere than to changes in the ozone concentration that might reasonably be expected to occur.

6. Accuracy of results

At every stage of these calculations, various checks have been made of the accuracy of the results. In the first place, from an estimate of the experimental error for the laboratory determination of the transmission function, τ, it is possible to determine directly from Eqs. (1) and (2) the corresponding error in the calculated values for the upward and downward flux. A liberal estimate of the experimental error of Summerfield's measurements is 5 per cent. The corresponding error in the upward and downward flux is about 3 per cent. This is smaller than the original experimental error since the black-body flux that enters into Eqs. (1) and (2) is known exactly. However, it is more difficult to estimate the error in the cooling rates since these depend essentially on the determination of the derivative of the transmission function.

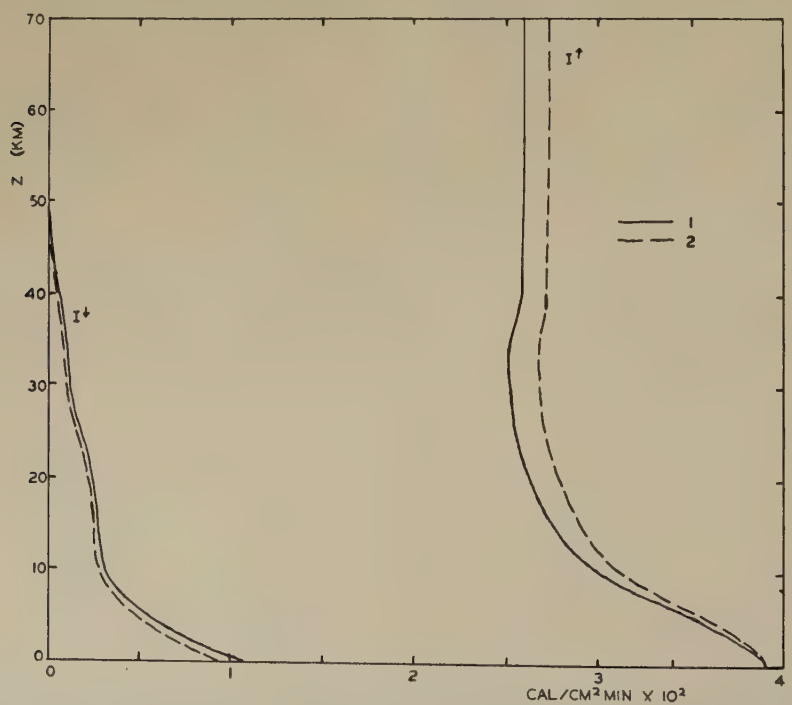

Figure 7. The upward and downward radiation flux ($I^{\uparrow}$ and $I^{\downarrow}$ respectively) in the region of the 9·6 μ ozone band for ozone-distribution curve L, temperature curve A, and transmission curves 1 and 2.

For this reason the calculations were repeated using the ozone distribution curve L and the temperature curve A with two different absorption curves, 1 and 2. Absorption curve 1 (Fig. 1) was used in all the previous sections of this article and is the best fit to the laboratory data. Curve 2 was drawn farther from curve 1 than the limits of the experimental error. The slope of both curves must be one-half near the left-hand side of Fig. 1 because this is the square-root region where the lines do not overlap. The slope of one-half was continued to larger path lengths for curve 2 than for curve 1. These curves also have different slopes for all path lengths beyond the square-root region. For large absorptions curve 2 gives 5 per cent less absorption than curve 1. This difference between the curves increases as the absorption become smaller and the difference is about 30 per cent when the absorption is 10 per cent. It is felt that the difference between the results computed for curves 1 and 2 gives an upper limit to the possible error in the final results that can be caused by errors in the determination of the transmission function.

The upward and downward flux is shown in Fig. 7. The differences between these two curves are relatively small. For example, the error in the determination of the upward flux is nowhere greater than 6 per cent, even though the transmission curves differ over part of their range by much more than this amount.

The heating and cooling rates are shown in Fig. 8. These are more sensitive to the transmission function, as was to be expected. At the upper levels, the results from curve 2 are as much as 30 per cent below those of curve 1. However, it is interesting to note that both curves have the same qualitative shape with their maxima and minima at the same altitude, even though their magnitudes are somewhat different. These features of the cooling curve are primarily determined by the temperature distribution and so are not sensitive to the values adopted for the transmission function.

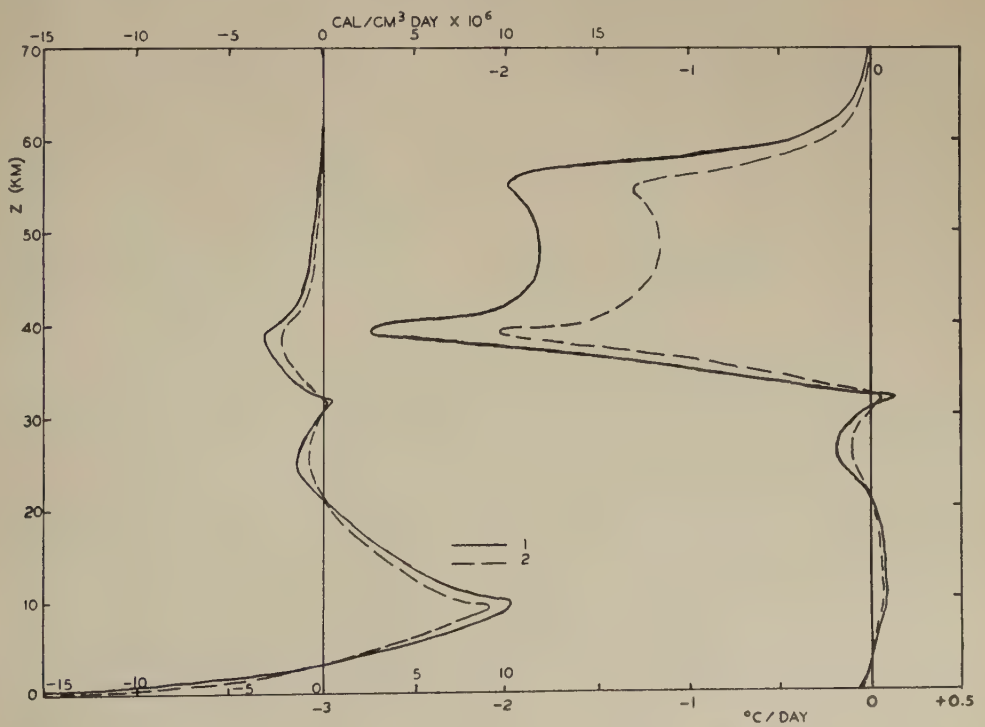

Figure 8. The heating or cooling rates for ozone-distribution curve L, temperature curve A, and transmission curves 1 and 2. The three curves at the left-hand side of the figure give the energy absorbed in units of cal cm^{-3} day^{-1}. The three curves at the right-hand side of the figure give the heating rate in °C/day.

Further tests to determine the accuracy of these results, in addition to those described above, have been made with the aid of the electronic computor. The estimated error deduced from all of these tests and based on the assumption that Summerfield's measurements are accurate to within 5 per cent is that : (a) the curves for the upward and downward flux are accurate to within 5 per cent; (b) the heating and cooling curves have an estimated error of 10 per cent below 20 km with the error increasing to 30 per cent at 50 km. Above 60 km the absolute value of the cooling rate in °C/day is rather uncertain. However, it is believed that the relative values of the different curves have been determined with considerably greater accuracy than this, since all the corrections have been made in the same manner for each of the curves.

7. Temperature changes at the surface of the earth

The temperature change at an absorbing surface at a given height in the atmosphere as the ozone concentration or temperature curves change can be many times larger than the corresponding change in the free atmosphere. The change in the equilibrium temperature at the surface of the earth, as the ozone concentration changes from curve L to M, can be calculated from the results shown in Fig. 3. The difference in the upward and downward flux of the radiation for these two curves is 0·0068 cal cm^{-2} min^{-1} at the surface of the earth. An appropriate calculation that makes allowance for the carbon dioxide and water vapour absorption in the infra-red, shows that the surface emits an additional 0·0033 cal cm^{-2} min^{-1} to space for each degree that the surface temperature rises. Thus the equilibrium temperature would rise 2·1°C if the ozone distribution changed from curve L to M and nothing else changed in the atmosphere. It is assumed that this small temperature rise does not introduce any new physical factors that can remove heat from the surface.

If the ozone distribution changed from curve L to N, the corresponding rise in the equilibrium temperature at the ground is 0·7°C. A temperature rise of the same order of magnitude is obtained for the top of cloud banks. Thus, if the ozone concentration increases in the troposphere owing to convergence and subsidence, or from other causes, the equilibrium temperature at the surface of the earth can increase by as much as several degrees Celsius through the action of the infra-red radiation processes alone. It would be difficult to observe this effect, since larger temperature changes would usually be caused by other meteorological factors in situations where the ozone concentration changes by a large amount. It is interesting to speculate that the average surface temperature would have to rise by a small, but measurable amount, if a change in the circulation pattern or some other factor should change the ozone distribution curve so that, on the average, its maximum would occur at lower altitudes. The temperature changes calculated here, which are caused by radiative processes, would be superposed on those from other meteorological factors; the latter factors may be considerably larger than the former but any complete discussion of climatic change must include the radiative factors. The possibilities of interrelation of solar phenomena with ozone distribution and the circulation pattern are very numerous (Craig 1950, 1951).

A similar, but opposite effect exists for water vapour. Since the concentration of water vapour usually decreases rapidly with height, convergence and subsidence would reduce the water vapour at lower levels. Thus the equilibrium temperature at the surface of the earth would be reduced owing to infra-red radiation effects. The change due to water vapour would oppose the ozone effect calculated above, and might on occasions be considerably larger. Too little attention has been given, as yet, to the changes in the infra-red flux as the water vapour mixing ratio changes with height.

8. Absorption of solar radiation; the 14 μ band

In order to obtain a complete picture of the action of ozone on the cooling rate in the atmosphere, the direct absorption of solar radiation by ozone and the influence of the 14 μ band must be considered. There are, as yet, no extensive laboratory measurements of the absorption due to the 14 μ band and it is not possible to make calculations similar to those given here for the 9·6 μ band. The effect of this additional band would be to increase somewhat the magnitude of the cooling rates given here. However, this increase would probably be small, since the total intensity of this band is several orders of magnitude smaller than the intensity of the 9·6 μ band and since it is shielded to some extent by carbon dioxide. However, it should be determined whether the carbon dioxide does effectively shield this band at all altitudes. At the higher altitudes where the ozone concentration is large and the carbon dioxide band is no longer opaque at its centre, the 14 μ ozone band may make a small contribution to the cooling rate.

Recent calculations of the direct absorption of ultra-violet light from the sun by the ozone (Johnson 1953; Pressman 1955) show that the maximum diurnal temperature change is from 5°C to 10°C at heights of 40 to 50 km. Johnson's Fig. 5 is very similar to the results given here for the infra-red cooling above 35 km. His heating rates are approximately twice the infra-red cooling rate for ozone curve L and temperature curve A. The heating due to ultra-violet absorption could be balanced by the cooling due to infra-red radiation from 40 to 60 km for a temperature curve with values intermediate between our curves A and C. Since carbon dioxide can contribute to the cooling rate at these altitudes, it did not seem worth while to make such a calculation until the carbon dioxide cooling rates are better known.

Recent calculations of the cooling rate due to water vapour (Möller 1951, London 1954) indicate that its maximum value is about 5°C/day and that such large values can only be obtained within a few kilometres of the earth's surface, or near the top of a cloud. Above 15 km the cooling rate due to water vapour is less than $\frac{1}{2}$°C/day and decreases with altitude. Thus the influence of water vapour on the cooling rate, is considerably more important than that of ozone below 15 km. At greater altitudes, the ozone becomes more important than the water vapour. Above 30 km, the water vapour can possibly be neglected, unless it is found to occur in unexpectedly large concentrations at these altitudes.

9. Conclusions

The variation of the upward and downward infra-red radiation flux and cooling rate has been determined for three different ozone distributions and for three different temperature distributions. In all cases the infra-red radiation interacting with the 9·6 μ band acts to cool the atmospheric layer from the ground to several kilometres and to heat the atmosphere from there to approximately 20 km. The heating and cooling rates are never larger than a few tenths of a degree per day at these altitudes. The radiation cools the atmosphere by amounts of the order of 2°C to 3°C/day from 35 to 60 km. If this portion of the atmosphere is heated above its normal temperature, this cooling rate is greatly increased. The overlapping of the spectral lines together with the variations of their intensities and half-widths with pressure and temperature have been taken into account in this calculation. For a normal temperature and ozone distribution, the calculated infra-red cooling rate for ozone is approximately one-half the heating rate from ultra-violet absorption.

Acknowledgment

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Local temperature variations in the Reading area

By M. PARRY
Reading University

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Summary

The following paper describes some results of an investigation of local climate – based on a close network of meteorological stations – undertaken over an 18-month period in the Reading area. Mean and extreme temperatures, and temperature distributions on selected occasions are examined in relation to the local factors involved : these are mainly the relief and the presence of the town. In particular, the relation between weather conditions and variations in minimum temperatures is considered. Diurnal temperature variations at different sites are compared. Although the local relief is slight and the urban area neither closely built up nor heavily polluted, temperature differences of up to 8°F can occur, and differences in minima of 4°-5° are common, during radiation nights. Maximum temperatures vary little.

1. Introduction

Among the variety of local climates which exist within the framework of the broad regional climate, the urban climate deserves the interest of climatologists as an element of the physical background of town life, and as an artificial climate which may be modified by suitable planning. A full account of the nature of the urban climate has been provided by Kratzer (1937), and the bibliography has been brought up to date by Geiger (1950). The town climate can rarely be studied in isolation, as most urban areas are superimposed on natural surfaces of far from uniform character; the whole presents a composite problem of local climatology. Furthermore, urban development takes different forms, and the climate of an industrial town differs from that of a residential town or garden city or seaside resort. There is value, then, in investigating the nature and magnitude of local climatic variations in particular combinations of urban form and underlying relief. Such a quantitative survey was undertaken in the Reading area between June 1951 and November 1952, and this paper is concerned with the local variations in temperature observed during that period.

2. The Reading area

Reading is a town of moderate size (some 120,000 people) occupying an area of slight though definite relief. As indicated by the map and profile in Fig. 1. the relief pattern is dominantly that of a series of level terraces (formed largely of superficial deposits) rising in steps from the broad Thames flood-plain.

These surfaces are, in ascending order, the alluvial flats of the rivers (at 120-130 ft): gravel terraces south of the Thames at 140-150 ft and at about 200 ft, but only a single intermediate terrace at 170-180 ft north of the river : and finally, higher plateau gravel stretches at heights between 250 and 330 ft. These plateaux and terraces are cut by the major river valleys into three blocks. North of the Thames flood-plain, the Caversham block is the southern extension of the main Chiltern Plateau (which rises eventually to over 800 ft north of Nettlebed). South of the Thames, the Earley and Tilehurst blocks are separated by the narrow gap cut by the Kennet, and elsewhere practically surrounded by extensive flood-plain. The range of altitude is modest – about 150 ft separates the lowest and highest stations – but the pattern of relief is well-defined. The Thames itself is here, in places, up to 100 yd wide.

Fig. 1 shows also that the main extension of the built-up area is on the lower gravel terraces. The higher parts are distinctly suburban in character and, generally, away from the relatively small heavily built-up nucleus, the urban pattern is fairly open and the proportion of green space to total area is high. There are few blocks of flats, industry is mainly light, and there is little of what may be called industrial wasteland. Atmospheric pollution is not a serious problem. The urban factor cannot therefore be as strongly marked as in the larger, more closely built-up and more heavily polluted towns.

3. The form of the investigation

The investigation was based on data derived from a close network of orthodox meteorological stations. The results are therefore comparable with those of similar studies in the Nottingham area (Tinn 1938), the Manchester area (Manley 1944), and the Bath area (Balchin and Pye 1947, 1949-50). They are less strictly comparable, however, with most North American work in this field (the urban stations, there, being nearly always on roof-top sites), or with the results of much Continental work based on mobile surveys with portable equipment. The use of meteorological stations in a local study has many advantages – the problem is discussed by Balchin and Pye (1947) and by Sundborg (1951) – but it is an essential requirement of this method that the siting of the stations should be as truly representative as possible of the different facets of the area.

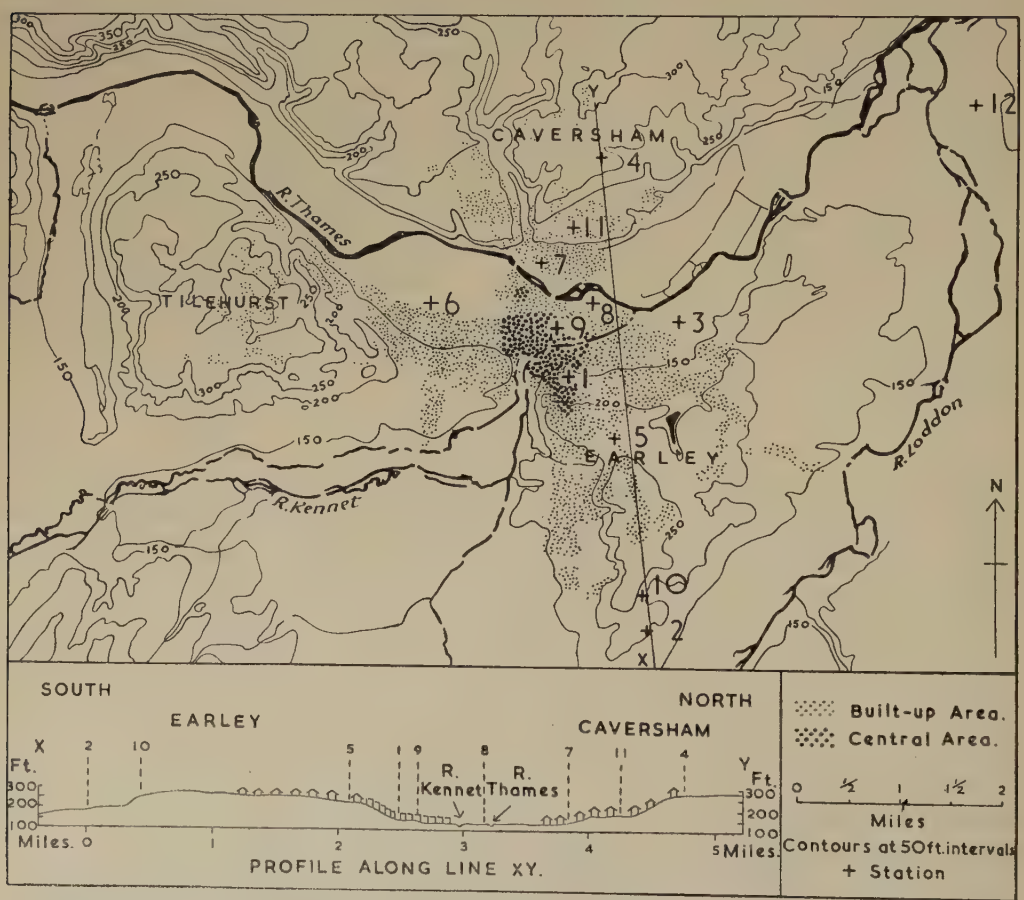

Figure 1. The Reading area : relief, extent of built-up area, and location of meteorological stations. Numbers refer to stations listed in Table 1.

Twelve stations were used in the Reading survey (three being permanent auxiliary climatological stations), and their distribution is shown on Fig. 1. The station numbers correspond with those of Table 1, to which reference should be made for details of heights of stations and types of exposure. Nine of the stations were situated on or very near the north-south profile shown on Fig. 1, a line that traverses all the major terraces and the different types of urban pattern. At all stations but one, conditions of exposure, while varying with the nature of the station, conformed with the requirements laid down by the Meteorological Office. The exception was station 9, which was somewhat over-sheltered by buildings and trees, an exaggerated urban exposure not inappropriate for the town centre. The other urban stations were sited on small lawns surrounded by buildings, the suburban stations on larger lawns more freely exposed, and the remainder on sites as open as possible.

At each station, observations were made once daily at 09 GMT (09 hr clock time at station 9), and the procedure in general corresponded to that at an auxiliary climatological station. The thermometers used were either certified or standardized against certified thermometers. Records of daily temperatures were maintained continuously at ten stations : stations 11 and 12 operated for most of the period.

4. Mean and extreme daily temperatures

For each station that operated continuously, averages were obtained of daily maximum, minimum and mean temperatures for each month. In Table 1 these mean values (all readings are in Fahrenheit) are averaged for the whole period and for each season (the usual climatological definition of the seasons being adopted), and are expressed as differences

TABLE 1. Mean daily temperatures (°F), July 1951-November 1952 (expressed as differences from values at station 1)

Station No.	1	2	3	4	5	6	7	8	9	10	11*	12*
Elevation (ft)	152	200	141	274	230	141	125	120	143	248	178	124
Exposure	Urban	Open	Open	Open	Suburban	Open	Urban	Open	Urban	Open	Suburban	Open
						(a) Daily mean temperatures						
All months	52·1	− 0·9	− 0·6	− 1·0	− 0·9	− 0·7	− 0·3	− 0·5	+ 0·7	− 1·2		
Summer	62·4	− 1·0	− 0·6	− 1·1	− 1·0	− 0·8	− 0·3	− 0·5	+ 0·7	− 1·3		
Autumn	50·0	− 0·8	− 0·6	− 0·8	− 1·0	− 0·8	− 0·2	− 0·3	+ 0·6	− 1·1		
Winter	40·1	− 0·9	− 0·3	− 1·0	− 0·6	− 0·7	− 0·4	− 0·5	+ 0·6	− 1·1		
Spring	51·5	− 1·1	− 0·8	− 1·0	− 0·9	− 0·6	− 0·3	− 0·9	+ 0·9	− 1·4		
						(b) Daily maximum temperatures						
All months	59·3	− 0·5	− 0·1	− 0·7	− 0·6	− 0·4	+ 0·1	0·0	+ 0·8	− 1·0		
Summer	70·9	− 0·5	− 0·4	− 0·8	− 1·0	− 0·5	− 0·1	0·0	+ 1·3	− 1·1		
Autumn	56·5	− 0·3	− 0·2	− 0·6	− 0·6	− 0·6	+ 0·1	+ 0·4	+ 0·5	− 1·0		
Winter	45·4	− 0·5	+ 0·3	− 0·8	− 0·1	0·0	+ 0·1	− 0·2	+ 0·3	− 0·7		
Spring	59·3	− 0·6	− 0·2	− 0·5	− 0·5	0·0	+ 0·2	− 0·1	+ 1·1	− 1·1		
						(c) Daily minimum temperatures						
All months	45·0	− 1·4	− 1·1	− 1·3	− 1·2	− 1·2	− 0·7	− 1·1	+ 0·7	− 1·4		
Summer	53·8	− 1·6	− 1·1	− 1·5	− 1·3	− 1·3	− 0·8	− 1·0	+ 0·5	− 1·6		
Autumn	43·4	− 1·2	− 1·1	− 1·0	− 1·2	− 1·0	− 0·6	− 0·9	+ 0·8	− 1·1		
Winter	34·9	− 1·3	− 0·8	− 1·2	− 1·0	− 1·2	− 0·7	− 0·7	+ 1·0	− 1·5		
Spring	43·8	− 1·7	− 1·5	− 1·6	− 1·3	− 1·5	− 0·9	− 1·6	+ 0·6	− 1·8		

* Records not continuous

from the values recorded at the urban station 1 (University). The variations in daily mean temperatures are small but not without significance. The largest differences (about — 1°F, for all months) are shown by the highest stations having open or rural exposures (except for station 5 which is suburban). Both the difference in altitude and the difference in exposure seem to be involved here. Stations of open exposure but at heights similar to that of station 1 are about ½°F colder on average. This ½°, then, represents broadly the contrast between urban and rural exposures, and the remaining ½° represents the contrast in altitude : this corresponds broadly with the normal lapse rate of 1°F in 300 ft. The greatest differences between stations occur in spring and summer.

If mean daily maximum temperatures are considered in the same way, the differences are seen to be very small, but the plateau stations again have the lowest values and the urban stations (especially 9) the highest. The greatest local differences occur at night. The plateau stations record, on average, minima nearly 1·5°F below the values at station 1 (all months considered), while ' rural ' stations at roughly the same height as station 1 have minima about 1° lower : here the urban factor operates alone. Station 9, representing the closely built-up town centre, is always the warmest station at night : of the three urban stations, 7, situated on the valley floor, is always the coldest.

When extreme temperatures were examined, it was soon evident that the extreme maxima at the various stations differed only slightly from each other, and from the corresponding mean maxima. Among the extreme minima, however, departures of — 4°F from the station 1 values were common, especially in the summer months. The urban stations nearly always registered the highest extreme minima. The lowest values were recorded not always by the plateau stations, but often by the valley-floor and low terrace stations with open exposures : here is an indication, not apparent in the mean values, of the significance of temperature inversions in the local climatology.

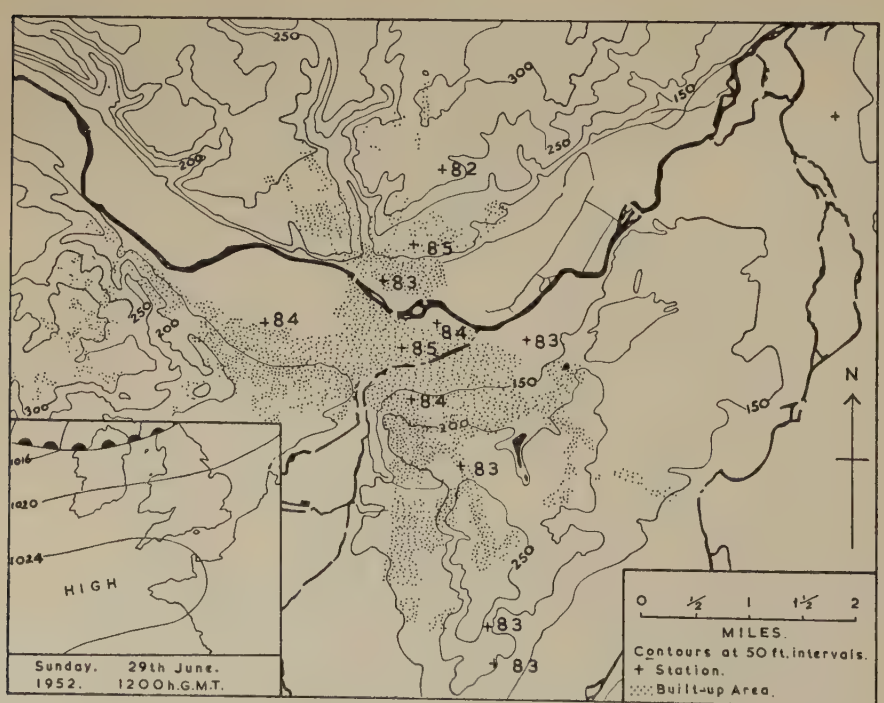

Figure 2. Maximum temperatures, 29 June 1952.

5. Selected temperature distributions

In Figs. 2-5 is presented a selection of characteristic distributions of maximum and minimum temperatures on days of widely varying weather conditions : on each map, the values are plotted against a background of the relief and the built-up area, and the inset map shows the accompanying synoptic situation.

(*a*) *Maximum temperatures*

Fig. 2 shows the distribution of maximum temperature on a fine summer day with little cloud and wind. Even under these conditions, which most favour the development of local contrasts, the differences between stations are small, the range of values being 3°. The plateau stations are a degree or two cooler than the valley stations, and there is a slight urban effect. Station 11 evidently benefits from a favourable aspect on a south-facing terrace.

Under cloudy, windy conditions, the urban influence vanishes and the differences generally amount to no more than a degree. Winter day-time temperature inversions did not occur during the period. All the evidence, in fact, indicates that the local influences are rarely strong enough to assert themselves against the turbulence which normally reaches its peak in the afternoon and thus tends to impose a general uniformity of maximum temperature throughout the area.

(*b*) *Minimum temperatures*

On the other hand, the distributions of minimum temperature show two contrasted patterns, associated with cloudy, turbulent conditions and with anticyclonic conditions respectively (though intermediate types are well represented). On nights with much cloud and wind, differences between minima remain small. Fig. 3 shows a pattern that

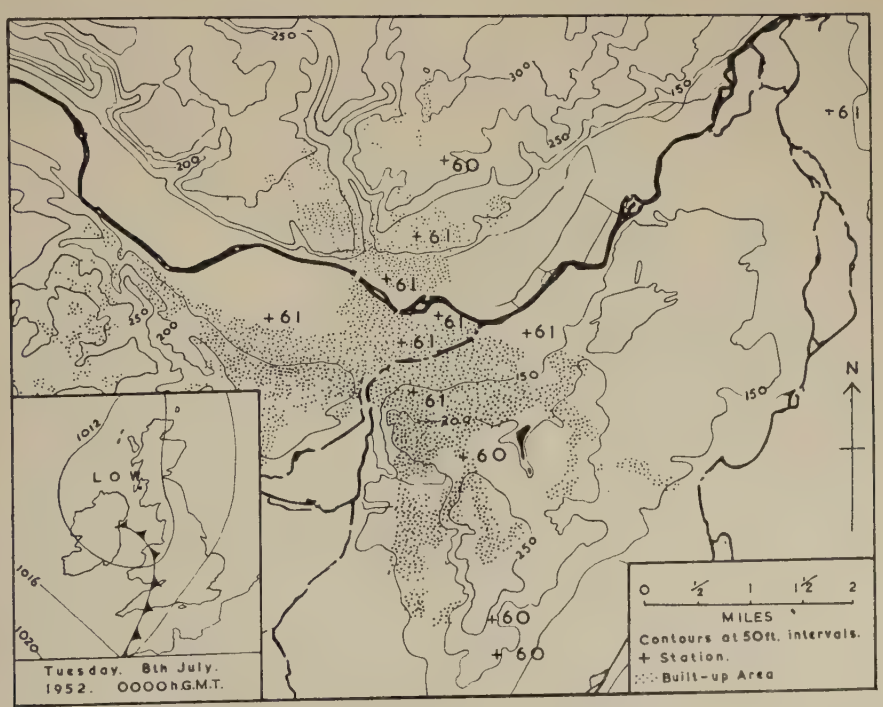

Figure 3. Minimum temperatures, 8 July 1952.

occurred under conditions of increasing frontal cloud and winds of Force 3 to 4 : the only significant local factor is relief, the plateau stations being a degree cooler than sites in the valley. This difference corresponds broadly with the dry adiabatic lapse rate (1°F in 180 ft) which must in fact be established in the lower air layers under such turbulent conditions.

Considerable temperature contrasts may develop, however, on the clear calm nights often associated with anticyclonic situations. The example illustrated in Fig. 4 is quite typical of such radiation nights in spring, summer and autumn : the winter distributions are similar but the range of differences is generally less. Both the relief and the urban factor are significant in this example. The temperature inversion is well developed, the low-lying stations away from the town being up to 4°F colder than the plateau stations. In the town centre, the urban warmth is sufficient to destroy the surface inversion, though station 7, with a valley-floor site in the less densely built-up Lower Caversham area, while warmer than valley stations away from the town, is nevertheless colder than the plateau station 4 : the urban influence is here weaker than that of relief.

On a few occasions a complete inversion occurred, undisturbed by the urban warmth. Such a pattern was recorded on an October night (Fig. 5), part of a spell of bright cloudless weather with radiation fog developing at night. Not only did this example provide the most marked inversion observed during the period – the plateau station 4 recording a minimum 8°F above that of the valley station 12 – but the urban stations 1 and 9, normally the warmest, recorded minima 2°F below that of station 4. This is an instance of a very complete domination of the urban factor by that of relief.

These examples show that the most marked temperature contrasts occur under conditions of strong outgoing radiation, and illustrate the typical operation of the local factors involved : these are plainly the relief and the urban influence (which is usually, though not invariably, the dominant of the two). On a number of radiation nights, no

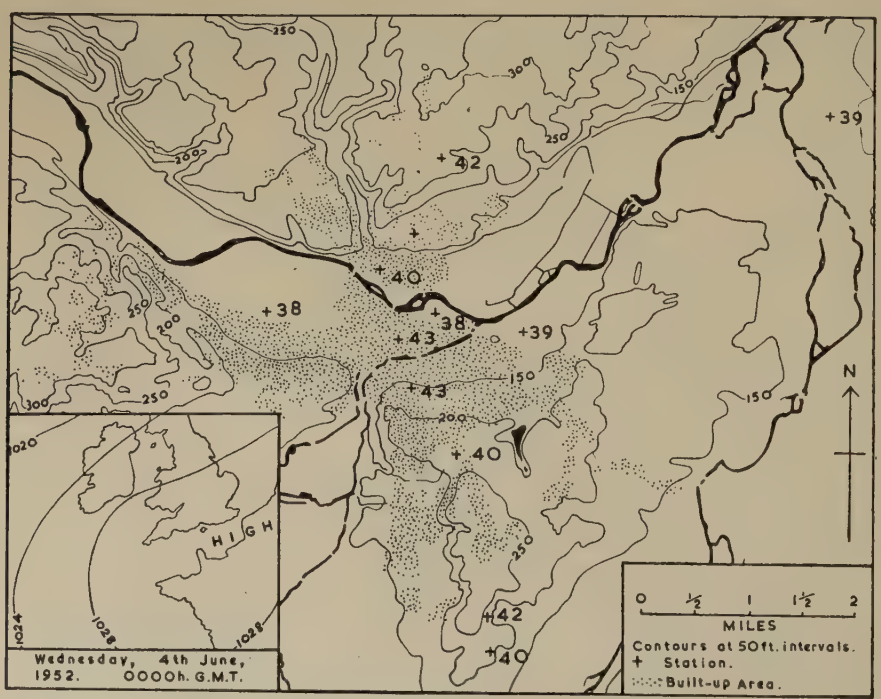

Figure 4. Minimum temperatures, 4 June 1952.

inversion was apparent, plateau stations being as cold as valley stations, although the urban effect was well marked. These were probably cases of deep inversions, undetected because of the lack of stations higher up on the Chiltern Plateau. The small range of altitude in the Reading area, and its general setting near the western end of the London Basin, must not be forgotten. On such occasions – similarly when a deep radiation fog submerges the whole of the Reading area – the local climate is overpowered by the regional climate, i.e., the accumulation of cold air in the London Basin as a whole. It is also of interest that stations with open exposure situated within or near the built-up area (6 and 8) behave like more distant rural stations in recording low minima on radiation nights. This testifies to the comparative cleanliness of the Reading atmosphere : a typical smoke pall would reduce nocturnal radiation from these sites and maintain higher minima.

6. Minimum temperatures and nocturnal weather conditions

Since the weather at the time is clearly a significant permissive factor in the development of local variations, special attention has been paid in this work to the problem of relating patterns of minimum temperatures to the prevailing conditions of cloudiness and wind. In the absence of any observations made at night at the Reading stations, use was made of the *Daily Weather Report* observations for the station nearest Reading, which is London Airport (21 mi distant), assuming conditions there to be reasonably representative of those in the Reading area. On this basis, each night was classified according to state of sky, using the categories ' b,' ' bc,' ' c,' and ' o ' (where b and bc have their standard meanings, c signifies 6-7/8ths of cloud, and o completely overcast), and also according to wind force, using the Beaufort Scale. The minimum temperatures for each night, at each station, were expressed as departures from the corresponding values at station 1 (University), and these departures placed in the category of state of sky and wind force

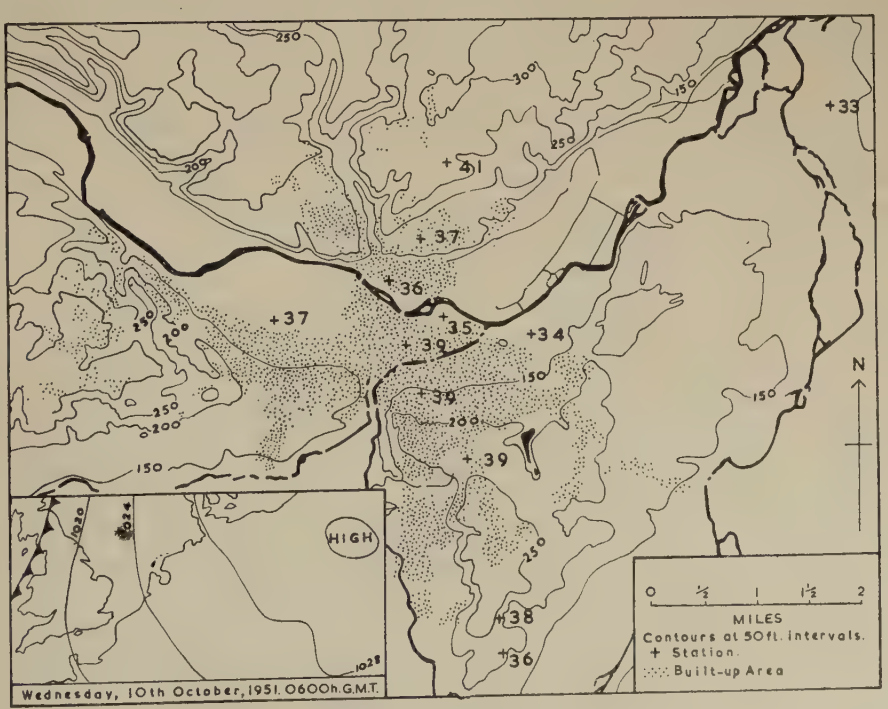

Figure 5. Minimum temperatures, 10 October 1951.

TABLE 2. MEAN DIFFERENCES IN MINIMUM TEMPERATURES BETWEEN UNIVERSITY AND OTHER STATIONS (SUMMER PERIOD)

Kings Meadow (120 ft, open exposure)

Wind / Sky	0	1	2	3	4	5 +
b	− 3·0	− 3·2	− 1·7	+ 1·0	0	
	8	13	10	3	1	
bc	− 2·0	− 1·7	− 0·8	− 0·5	− 0·5	0
	4	21	32	13	4	3
c	− 3·0	− 0·2	− 0·7	− 0·1	0	
	1	5	7	16	4	
o		− 1·7	− 0·3	− 0·3	+ 0·1	
		3	7	7	6	

Caversham Park (274 ft, open exposure)

Wind / Sky	0	1	2	3	4	5 +
b	− 1·7	− 2·8	− 1·8	− 1·0	− 1·0	
	8	13	10	3	1	
bc	− 2·5	− 2·0	− 1·6	− 1·3	− 1·0	0
	4	21	32	13	4	3
c	− 2·0	− 0·8	− 1·0	− 1·2	− 0·3	
	1	5	7	16	4	
o		− 1·3	− 1·0	− 1·3	− 1·0	
		3	7	7	6	

Lower Caversham (125 ft, urban exposure)

Wind / Sky	0	1	2	3	4	5 +
b	− 1·5	− 2·1	− 1·2	− 0·5	0	
	8	13	10	2	1	
bc	− 1·3	− 1·2	− 0·8	− 0·3	− 0·7	+ 0·3
	4	21	32	13	4	3
c	− 1·0	0	0	− 0·1	− 0·3	
	1	5	7	16	4	
o		− 1·0	− 0·4	− 0·4	− 0·1	
		3	7	7	6	

Forbury Gardens (143 ft, urban exposure)

Wind / Sky	0	1	2	3	4	5 +
b	+ 1·0	+ 0·5	+ 0·5	+ 0·3	+ 1·0	
	8	13	10	3	1	
bc	+ 1·0	+ 0·8	+ 0·4	+ 0·5	0	+ 0·7
	4	21	32	13	4	3
c	+ 1·0	+ 1·0	+ 0·6	+ 0·8	+ 0·3	
	1	5	7	16	4	
o		+ 1·0	− 0·3	+ 0·1	− 0·5	
		3	7	7	6	

appropriate to that particular night. Finally, all such figures in each category were averaged, the result being a series of tables as in Table 2, in which, for each station, mean differences in minimum temperature were shown for various conditions of sky and wind. The data were analyzed on a seasonal basis, so that four tables were constructed for each station.

Table 2 gives four examples, all for the summer period (chosen because it combines two summer seasons), for selected stations of contrasted heights and exposures : for comparison, it should be remembered that the University station is at 152 ft and has an urban exposure. Since the reliability of the averages depends on the number of occurrences within each category, the number is given below each average.

In general the greatest departures occur in the top left-hand corner of the tables, i.e., on radiation nights, and the differences become more or less progressively reduced away from that corner. At Kings Meadow (station 8), the freely exposed valley-floor station, minima are about 3°F lower than the University averages on clear calm nights. This is mainly the difference between urban and rural exposures. Under cloudy and – especially – windy conditions, the differences on average are small : with wind Force 4 and above, they are negligible, which implies that at such wind speeds the urban effect is practically eliminated. In the case of Caversham Park (4) on the open plateau, the largest differences again occur towards the top left-hand corner of the table, though not in the b0 category, owing perhaps to the frequency of inversions on such nights. Under windy turbulent conditions, there is a difference of about 1°F from the University, representing the difference in elevation (122 ft) between the two stations. At Lower Caversham (7), the low-lying urban station, conditions differ from those at Kings Meadow in the smaller departures on radiation nights. Finally, Forbury Gardens (9), the over-sheltered town centre station, is slightly warmer than the University under practically all conditions.

This method is clearly not here used to the best advantage, owing to insufficient occurrences and uncertainties involved in the use of the London Airport observations. Nevertheless it demonstrates sufficiently the operation of the local factors under various weather conditions. Tables for the other seasons show similar though less well defined features.

The relationships between minimum temperatures and nocturnal conditions may be considered also in terms of the most frequently occurring departures between the University and other stations. Frequency distribution graphs were drawn for all stations : Figs. 6

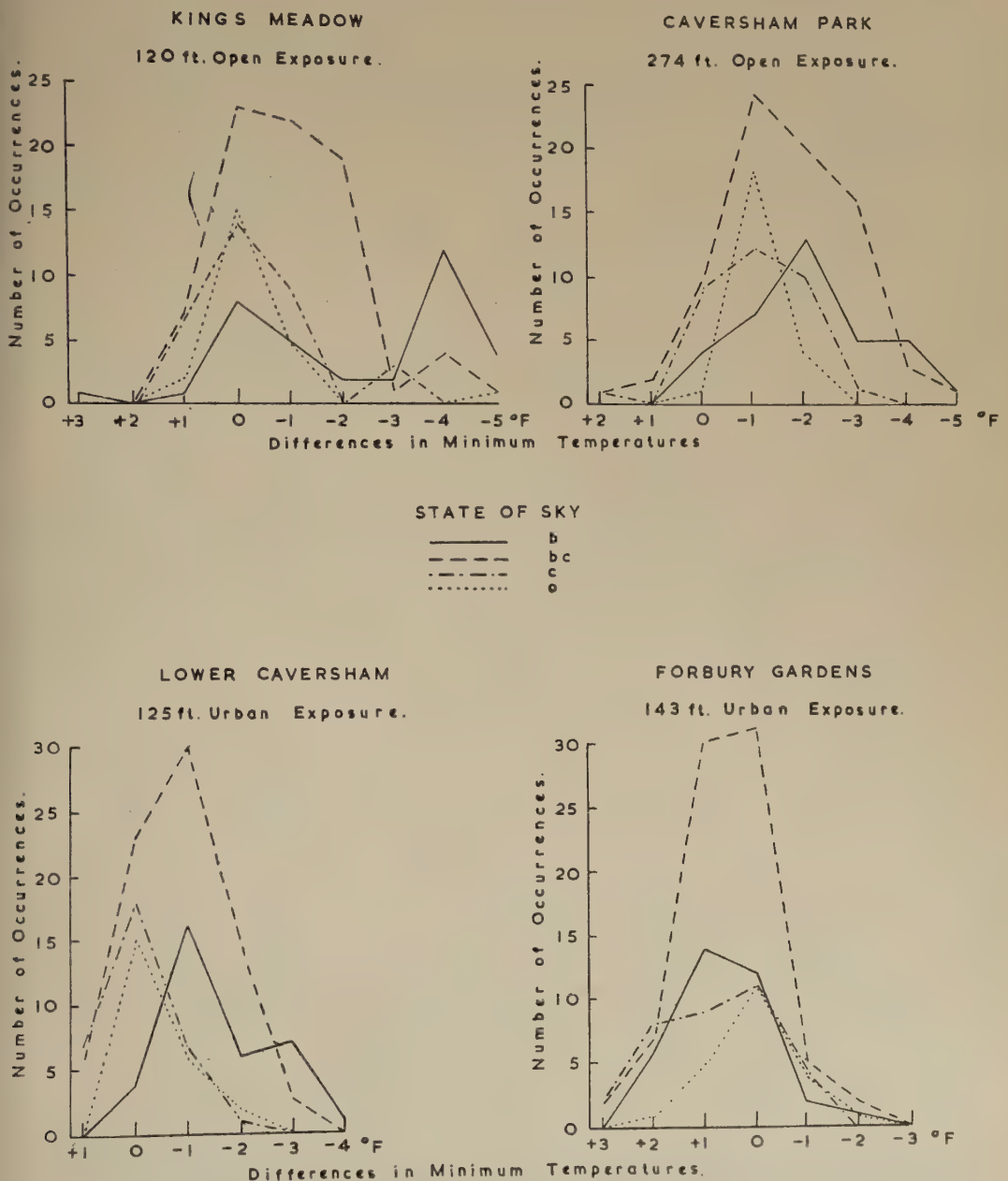

Figure 6. Frequency distributions of differences in minimum temperatures between University and other stations, in relation to state of sky. (Summer period).

and 7 provide examples for the four stations previously selected, again for the summer period. The horizontal axis of each graph represents the difference in minima from the University values, and the vertical axis the number of occurrences of these differences.

In Fig. 6, frequency curves have been plotted for occurrences within each of the categories of state of sky. The Kings Meadow curves show that on clear nights this station is most frequently 4°F colder than the University, but with increasingly clouded skies the peak shifts to 0°F. This clearly demonstrates the suppression of the urban influence on cloudy nights when outgoing radiation is at its least. Caversham Park shows a similar displacement of the peak, which however stops at — 1°F, even under overcast conditions, i.e., the effect of exposure is overcome but that of relief remains. On clear nights, the Caversham Park minima are most frequently 2°F lower than those of the University, compared with 4°F for Kings Meadow : this again demonstrates the inversion effect. In the curves for Lower Caversham and Forbury Gardens the movement of the peak is slight.

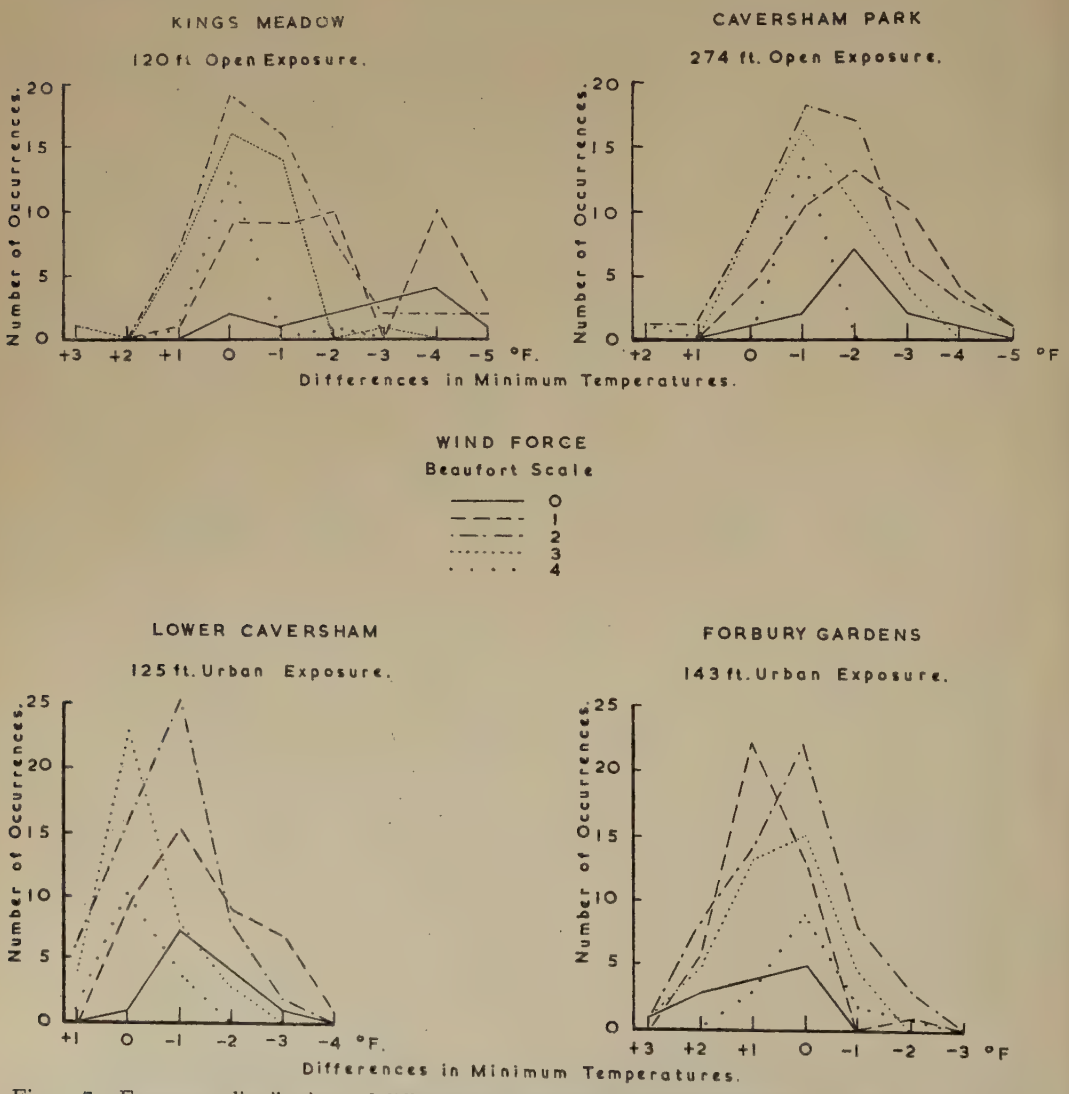

Figure 7. Frequency distributions of differences in minimum temperatures between University and other stations, in relation to wind force. (Summer period).

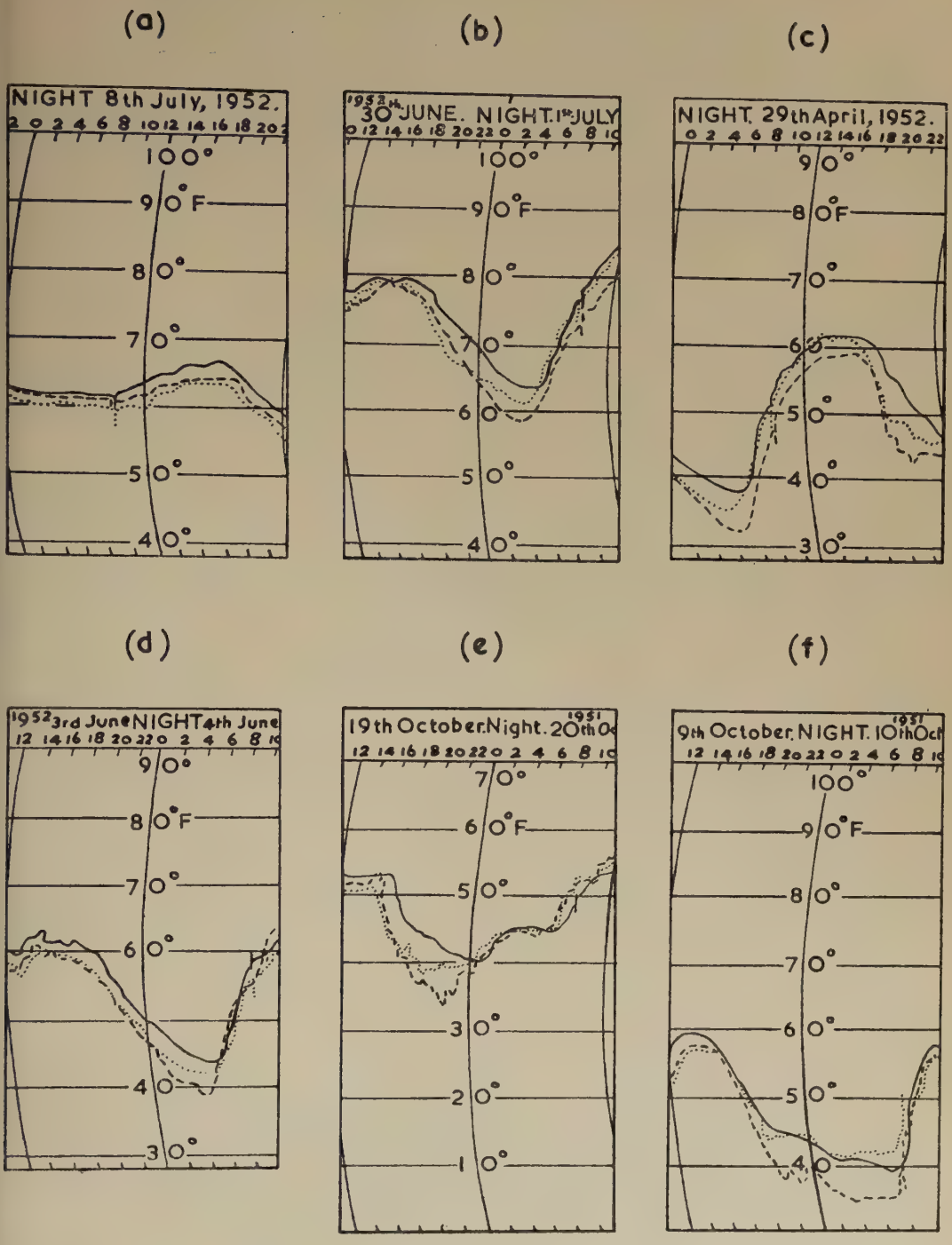

Figure 8. (a)-(f) Superimposed thermograph traces for University (solid line), Kings Meadow (dashed line) and Caversham Park (dotted line), for selected periods.

In these graphs the factor of wind force has been ignored, and consequently there are apparent anomalies : for example, the secondary peak at 0 in the b curve for Kings Meadow. Similar graphs relating to five categories of wind force on the Beaufort Scale complete the general picture (Fig. 7). These curves also show the movement of the peak frequency from large to small differences, this time as the wind force increases. The evidence of the Kings Meadow curves reinforces the already stated view that with a Force 4 wind (and sometimes with a Force 3) the urban warmth is entirely eliminated. In the case of Caversham Park, the shift of the peak stops at $-1°F$, and this is clearly related to the difference in elevation between the plateau station and the University, and to the lapse rate under turbulent conditions.

7. Continuous temperature records

Thermographs were maintained at three stations, Kings Meadow and Caversham Park (representing respectively open conditions on the flood-plain and on the plateau), and the University (at an intermediate elevation and representative of most of the urban area). Some changing temperature relationships between the three stations over selected 24-hr periods are illustrated by the superimposed thermograph traces of Fig. 8.

The first example (a) illustrates conditions on a cloudy, rather windy, summer morning, followed by a somewhat sunnier afternoon : the morning traces show a general uniformity, with, however, the Caversham Park temperature steadily a degree below the others (the relief effect), while in the afternoon the urban warmth becomes apparent. The remaining examples are of anticyclonic conditions, characterized by a considerable diurnal range of temperature generally, and marked differences in the individual traces which reflect clearly the local influences. There is in each case a well marked nocturnal inversion : on most occasions, this is overcome by the urban warmth, but included is the example, already referred to, of the complete inversion (f).

The diurnal range of temperature is smallest at the town station, owing mainly to the higher minima, and largest at Kings Meadow. The daily march of temperature within the town differs from that outside. The morning rise of temperature at the urban station proceeds more slowly than at rural stations more exposed to the low morning sun : on the other hand, the evening temperature fall is delayed in the town – owing to the retention of heat in brickwork, pavements, etc. – with the result that the largest temperature difference between town and country occurs in the evening (e.g., 8°F between University and Kings Meadow at about 21 hr in example (e)). It is again characteristic of radiation nights that the time at which minimum temperature occurs is later at the town station than at the other two (examples (b) to (f)).

8. Conclusions

The evidence presented shows that substantial local temperature variations may occur within an area in which the range of altitude is modest and the urban pattern relatively open. There seems no need to look to local factors other than the relief and the presence of the town in order to explain these temperature relationships. There is no evidence that the rivers are significant on a local – as opposed to a micro – scale. The flood-plain station Kings Meadow (situated 200 yd from the Thames) frequently recorded the lowest minima in the whole area, though records from a station on the river bank would perhaps have reflected the influence of the river. It was not possible to place every station on the same type of subsoil : apart from the flood-plain stations 7 and 8, all were sited on gravel, with the exception of 5 and 10, which were on clay. This too, seems to have no significance in the records.

The general control exerted by the prevailing weather over local temperature variations has been demonstrated. The analyses reveal that, for Reading, the thermal effect of the town is eliminated by a wind of Force 3-4 (say between 10 and 15 mi/hr). In comparison, Mey (quoted by Kratzer 1937) has put the significant wind speed for the larger town of Bremen at 8 m/sec (about 19 mi/hr, between Force 4 and 5). Finally, it is evident that the town of Reading exhibits a modified type of urban climate – that of a predominantly residential town – characterized less by the urban smoke pall (here weakly developed) than by the distinctive thermal properties of the urban surfaces and structures.

Acknowledgments

The author's thanks are due to the Director of the Meteorological Office for authorizing the loan of equipment, and to the Research Board of the University of Reading for the financial support, that together made this investigation possible. He must also express his gratitude to the many volunteer observers who co-operated enthusiastically in the maintenance of the records.

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551.524.7 : 551.547.3 : 551.510.5 : 551.515

Variation of temperature in the troposphere and lower stratosphere

By J. K. BANNON and A. GILCHRIST
Meteorological Office, Harrow, Middlesex

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SUMMARY

Correlation coefficients are evaluated between temperatures at 500 mb and at three levels in the lower stratosphere (highest 60 mb) and, respectively, the pressure at the tropopause and the height of the 300-mb level for Larkhill and Lerwick for each month of the year. Similar coefficients are also given for Malta and (for different levels) for Arctic Bay for January, April, July and October. The Dines type of relationship is confirmed for those regions and seasons affected by travelling depressions and anticyclones. There is no significant correlation between similar parameters for Aden and tropical islands in the Pacific. The relation between temperature in the lower stratosphere and, respectively, the level of the tropopause and the height of the 300-mb level decreases by a factor of 2 or more between the levels of 150 mb and 60 mb at Larkhill, Lerwick and Malta, indicating a marked decrease with height in the effect of tropospheric disturbances on stratospheric temperatures. The stratospheric correlation coefficients for Larkhill and Lerwick have a significant minimum in the winter.

1. INTRODUCTION

In temperate latitudes there is a well-known relationship between upper-air temperatures and the general pressure distribution and level of the tropopause respectively. A cold troposphere is associated with low pressure in the upper troposphere and a low height of the tropopause, the stratosphere being correspondingly warm; a warm troposphere occurs with high pressure in the upper troposphere, a high level of the tropopause and a cold stratosphere above it. The relationship is partly demonstrated by the correlation coefficients of Dines (1919) and is described in many textbooks.

Dines's correlation coefficients give quantitative evidence of the general model for the troposphere and his discussion gives qualitative evidence for the lower stratosphere. Ballon-sonde observations from a number of stations throughout Europe were all used together in the evaluation of the coefficients. The approximate magnitudes of these coefficients have been confirmed by Priestley (1945) for southern England. Corresponding correlation coefficients have not been evaluated for the lower stratosphere.

Bannon (1954) has discussed the above relationships in connection with a classification of upper-air temperature according to the pressure of the tropopause. Sawyer (1954) has also shown that advection and vertical motion are of equal importance in causing changes in the height of the tropopause, other factors, such as dissipation and re-formation, being much less important.

It has been pointed out by several writers (e.g., Riehl 1950) that these relationships do not always hold at high latitudes, where the stratosphere may be relatively cold above a troposphere which is also cold for its locality. It is known also (see, e.g., Schumacher 1952) that, over the Antarctic in winter, temperature sometimes decreases with height up to all levels so far explored by radiosonde, and there is no stratosphere. The temperature of the upper air in the Tropics has small variation and there is little evidence that there is a systematic variation with pressure distribution or tropopause level.

There is thus a need for quantitative test of the model of upper-air temperature variation, for two reasons, namely,

(i) to find how far into the stratosphere the model is applicable;
(ii) to find for what range of latitudes the model is applicable and its relative merits for different latitudes and seasons.

The answer to (i) will supply, among other things, information as to how high into the stratosphere the effects of the disturbances of the troposphere (depressions and anticyclones, etc.) are noticeable.

Accordingly, a series of correlation coefficients was evaluated for four stations; Arctic Bay (73°N, 85°18′W.), Lerwick (60°8′N, 1°11′W), Larkhill (51°11′N, 1°48′W), and Malta (35°50′N, 14°27′E). Observations from Aden (12°50′N, 45°1′E) and a number of Pacific stations within 20° latitude of the Equator were also scrutinized. The correlation coefficients are between the height of the 300-mb level (H_{300}) and temperatures at the 500-mb level (T_{500}) and at three levels in the stratosphere; also between pressure at the tropopause (P_T) and temperature at the same levels as above. For Arctic Bay the height of the 400-mb level was substituted for the height of the 300-mb level, as the tropopause is often below the 300-mb level at that station. These pairs of variables were suggested by the correlations of Dines between pressure at 9 km (P_9) and mean temperature 1-9 km (T_9) (correlation coefficient = 0·95), between P_9 and height of the tropopause H_c (0·84), between H_c and T_9 (0·79). Priestley (*loc. cit.*) has shown that T_{500} is a good approximation to the mean temperature of the troposphere for this statistical purpose. The height of the 300-mb level is approximately equivalent to the pressure at 9 km and the pressure of the tropopause was used as a measure of its height.

Two sets of correlation coefficients were calculated. In the first, data were analysed for a limited period for each station and for the months January, April, July and October. In these calculations the observations for different hours of the day were all processed together, and the sets of observations used for calculating coefficients for one level were not necessarily the same as those used for another; for example, the data for H_{300}, T_{500}, were not all for the same occasions as the observations used in calculating the coefficients for H_{300}, T_{60}. This analysis indicated that the correlations between both H_{300} and P_T and temperature at fixed pressure levels decrease with increasing height in the stratosphere. There was the possibility, however, that the smaller correlation coefficients at the levels of 100 and 60 mb might be partly caused by errors in the measurement of temperature at the higher levels arising from absorption of radiation by the instrument, though this was thought to be unlikely. Because of the importance of the result a second series of correlation coefficients was calculated for Larkhill and Lerwick for the stratospheric levels, separating day and night observations and also restricting the observations to those occasions when readings were obtained up to the 60 mb level. For this second series data for a much longer period were used and coefficients were evaluated for each month of the year.

2. Data

For the first series of calculations the following data for the four months January, April, July and October were used :

1. Larkhill, Lerwick – Observations made four times daily (03, 09, 15 and 21 GMT approximately) in the period 1948-50 inclusive.

2. Arctic Bay – Observations made at 03 and 15 GMT daily in the period 1948-50 inclusive. In January soundings frequently did not penetrate far into the stratosphere; for this month observations for the period 1947-51 were used.

3. Malta – Observations at 03 and 15 GMT daily for the period 1948-51 inclusive.

4. Aden and Tropical Pacific stations – Correlation coefficients were not computed, but observations were scrutinized for three years (1949-51) at Aden, and for certain months in 1949, 1950 and 1951 for the Pacific stations.

The data for Larkhill, Lerwick, Malta, and Aden are tabulated in the Upper Air Climatological Branch of the Meteorological Office. Data for Arctic Bay are recorded in the Meteorological Office Library on microfilm supplied by the Canadian Meteorological Service. The observations for the Pacific islands are available in the Daily Series Synoptic Weather Maps published by the U.S. Weather Bureau.

The pressure of the tropopause is tabulated for all the observations analysed except those for the Pacific islands; for these the tropopause was determined from the standard-level readings and the temperatures recorded at ' special points ' when these were given. Current Meteorological Office definitions (see, e.g., the ' Introduction to the Daily Aerological Record ') could be applied without difficulty to these tropical tropopauses, which are usually simple in structure. Selecting the level of the tropopause at British or Arctic stations is usually straightforward. At Malta, however, two tropopauses are commonly evident, except in summer when a high tropical tropopause is usually found at a pressure just less than 100 mb; the statistics refer to the lower tropopause when the tropopause is double.

For the second series of calculations all observations for Larkhill and Lerwick which reached the level of 60 mb during the period 1944-September 1953 were used for certain specified hours. Statistics were worked for each month of the year. Crawley (51°5'N, 0°13'W) replaced Larkhill as an observing station on 29 April 1953. Data from Crawley for the five months May-September 1953 were included with those for Larkhill.

TABLE 1. TROPOSPHERIC STATISTICS

	ρ_{H,P_T}	$\rho_{H,T}$	$\rho_{P_T,T}$	$\bar{H}\times10^{-1}$ (ft)	$\sigma_H\times10^{-1}$ (ft)	$\bar{P}_T$ (mb)	σ_{P_T} (mb)	$\bar{T}$ (°F)	σ_T (°F)	n
					January					
Larkhill	− 0·77	0·86	− 0·79	2970	77	237	52	− 9·8	8·5	175
Lerwick	− 0·77	0·85	− 0·78	2890	75	265	61	− 18·4	10·3	356
Arctic Bay	− 0·54	0·88	− 0·53	2123	35	298	44	− 45·0	7·8	191
Malta	− 0·72	0·87	− 0·69	3009	42	235	46	− 6·0	6·3	197
					April					
Larkhill	− 0·80	0·91	− 0·84	2980	67	257	61	− 8·8	9·4	169
Lerwick	− 0·71	0·86	− 0·72	2910	65	291	60	− 16·1	8·9	343
Arctic Bay	− 0·67	0·87	− 0·63	2175	57	327	47	− 40·0	9·2	161
Malta	− 0·78	0·87	− 0·69	3009	42	235	46	− 0·0	6·0	197
					July					
Larkhill	− 0·62	0·91	− 0·64	3008	44	226	34	+ 6·4	6·0	182
Lerwick	− 0·78	0·91	− 0·82	3030	51	245	35	+ 0·8	7·0	358
Arctic Bay	− 0·81	0·84	− 0·81	2332	41	272	44	− 9·0	7·5	165
Malta	− 0·16	0·69	− 0·18	3167	23	099	19	+ 19·0	4·2	142
					October					
Larkhill	− 0·80	0·91	− 0·72	3040	66	220	51	+ 0·6	9·3	181
Lerwick	− 0·83	0·86	− 0·74	2970	71	247	57	− 7·9	11·1	362
Arctic Bay	− 0·55	0·65	− 0·63	2184	36	310	38	− 33·0	7·9	93
Malta	− 0·35	0·79	− 0·45	3096	31	182	31	+ 7·0	4·4	203

Period of observation : Larkhill and Lerwick, 1948-50; Arctic Bay, 1948-50 but 1947-51 for January; Malta, 1948-51.

ρ_{H,P_T}, $\rho_{H,T}$, $\rho_{P_T,T}$ are correlation coefficients between H, P_T; H, T and P_T, T respectively.

H = height of 300-mb level, except at Arctic Bay where it is height of 400 mb; P_T = pressure at tropopause; T = temperature at 500 mb; σ_H = standard deviation of H; σ_{P_T} = standard deviation of P_T; σ_T = standard deviation of T; n = number of observations.

3. Results : troposphere

Correlation coefficients between H_{300} and P_T, H_{300} and T_{500} (H_{400} for Arctic Bay in each case) and P_T and T_{500} are given in Table 1 for the months January, April, July and October for each of the stations Larkhill, Lerwick, Arctic Bay and Malta. Other auxiliary statistics are also included in the Table.

There is no significant variation in the correlation coefficients for Larkhill and Lerwick throughout the year and they confirm the findings of Dines (1919) and Priestley (1945) for the troposphere. Table 2 shows corresponding values.

TABLE 2. Correlation coefficients compared

Dines (Europe)	$H_c\, T_m$	0·79	$H_c\, P_9$	0·84	$T_m\, P_9$	0·95
Priestley (Larkhill 1944)	$H_c\, t_{500}$	0·79				
Larkhill (1948-1950)	$P_T\, T_{500}$	− 0·75	$P_T\, H_{300}$	− 0·75	$T_{500}\, H_{300}$	0·90

Where H_c = height of the tropopause; T_m = mean temperature from 1-9 km; P_9 = the pressure at 9 km; t_{500} = the departure of temperature at 500 mb from the long-period monthly mean; and other symbols have the meanings described in Section 1. The figures for Larkhill are the means of corresponding figures for the four months January, April, July, October, separately.

At Arctic Bay the relation between the pressure of the tropopause and tropospheric temperatures is considerably less than over the British Isles except in July (Table 1, $\rho_{P_T, T}$, ρ_{H, P_T}).

At Malta in January and April the correlation coefficients are similar to those over Britain but in July and October they are small.

Statistics are not presented for Aden or for the Pacific stations. It is obvious that no significant correlation could be discovered from the data. It may be, however, that some relation will be found between upper-air temperatures and, for example, the pressure of the tropopause in the Tropics for the organized disturbances, easterly waves, etc., which occur. It is clear that there is little of the kind to be discovered in undisturbed weather in the troposphere.

4. Results : stratosphere

Correlation coefficients between the temperatures at three levels in the stratosphere and respectively the height of the 300-mb level (400 mb for Arctic Bay) and the pressure of the tropopause are given in Tables 3, 4, 5 and 6 for Larkhill, Lerwick, Arctic Bay and Malta. Other statistics necessary for the full interpretation of the correlation coefficients are also given in the Tables. The coefficients and other statistics for Larkhill and Lerwick (Tables 3 and 4) were worked for each month of the year. The three stratospheric levels considered were 150, 100 and 60 mb; the level of 150 mb is rarely in the troposphere over the British Isles. The data from which the correlations were computed were restricted to observations made either in darkness or within about three hours of midday so that the effects of direct radiation on the observing instruments can have introduced only small errors in the correlation coefficients. It would have been preferable to use only observations made in darkness but the number of observations reaching 60 mb is much less during darkness than during daylight* and in many months the number of observations made in darkness was insufficient for analysis.

* At the low temperatures experienced in the stratosphere, the elastic properties of the sounding balloons are improved by the direct heating by solar radiation and hence soundings in daylight usually reach higher levels than soundings in darkness.

Tables 3 and 4 were computed from observations which were complete up to the level of 60 mb. As a means of judging to what extent this selection may have introduced bias into the results some statistics computed from all observations made at Larkhill and Lerwick over a long period are given in Table 7. These unselected statistics can be compared with the similar statistics in Tables 3 and 4.

The statistics for Arctic Bay and Malta (Tables 5 and 6) were computed for the months of January, April, July and October and the data were not restricted to observations which reached the highest levels. At Arctic Bay the 250 mb level is usually in the stratosphere; observations for levels above 150 mb were seldom obtained in the period considered and the three stratospheric levels for which results were worked were therefore 250, 200 and 150 mb. At Malta the (lower) tropopause is seldom above the level of

TABLE 3. STRATOSPHERIC STATISTICS : LARKHILL

	ρ_{H,P_T}	T	$\rho_{H,T}$	$\rho_{P_T,T}$	$\bar{H}\times10^{-1}$ (ft)	$\sigma_H\times10^{-1}$ (ft)	$\bar{P}_T$ (mb)	σ_{P_T} (mb)	$\bar{T}$ (°F)	σ_T (°F)	n
Jan.	− 0·77	T_{150}	− 0·57	0·65	2990	64	218	49	− 76·8	10·6	112
(night)		T_{100}	− 0·51	0·49					− 78·4	6·7	
		T_{60}	− 0·24	0·25					− 80·3	7·5	
Feb.	− 0·78	T_{150}	− 0·59	0·72	2980	68	227	42	− 68·5	8·1	95
(day)		T_{100}	− 0·42	0·52					− 68·7	8·2	
		T_{60}	− 0·11*	0·36					− 67·4	10·0	
Mar.	− 0·65	T_{150}	− 0·62	0·76	3000	50	220	43	− 70·1	8·5	106
(day)		T_{100}	− 0·49	0·62					− 69·7	7·6	
		T_{60}	− 0·31	0·38					− 68·7	9·5	
Apr.	− 0·80	T_{150}	− 0·68	0·75	3010	58	230	45	− 63·3	7·0	116
(day)		T_{100}	− 0·62	0·61					− 64·0	5·9	
		T_{60}	− 0·21	0·37					− 61·8	5·8	
May	− 0·81	T_{150}	− 0·80	0·77	3040	53	235	39	− 59·1	8·2	119
(day)		T_{100}	− 0·74	0·67					− 58·1	5·9	
		T_{60}	− 0·42	0·32					− 54·1	5·8	
June	− 0·78	T_{150}	− 0·74	0·82	3070	47	217	36	− 59·8	8·6	120
(day)		T_{100}	− 0·69	0·68					− 58·4	5·8	
		T_{60}	− 0·46	0·49					− 53·6	5·8	
July	− 0·62	T_{150}	− 0·68	0·72	3100	38	214	28	− 59·1	7·4	111
(day)		T_{100}	− 0·68	0·59					− 59·0	6·4	
		T_{60}	− 0·41	0·43					− 52·8	6·3	
Aug.	− 0·79	T_{150}	− 0·81	0·76	3080	48	224	41	− 58·6	9·1	98
(day)		T_{100}	− 0·77	0·65					− 55·9	7·8	
		T_{60}	− 0·51	0·42					− 52·8	6·4	
Sept.	− 0·68	T_{150}	− 0·74	0·68	3070	46	215	42	− 66·1	8·7	120
(day)		T_{100}	− 0·65	0·49					− 65·8	6·9	
		T_{60}	− 0·42	0·30					− 60·4	6·4	
Oct.	− 0·80	T_{150}	− 0·84	0·80	3040	61	220	46	− 69·9	9·2	116
(day)		T_{100}	− 0·78	0·69					− 70·4	7·4	
		T_{60}	− 0·57	0·51					− 67·7	6·4	
Nov.	− 0·66	T_{150}	− 0·71	0·67	3000	57	220	51	− 75·1	9·6	79
(night)		T_{100}	− 0·63	0·54					− 77·1	6·6	
		T_{60}	− 0·21*	0·29					− 78·9	5·4	
Dec.	− 0·70	T_{150}	− 0·63	0·73	2970	65	220	44	− 74·8	9·3	83
(night)		T_{100}	− 0·53	0·58					− 76·6	6·3	
		T_{60}	− 0·09*	0·27					− 80·1	6·8	

Period of observations was 1944-53. Night observations were made at 00, 06 and 18 GMT 1944-47, at 03 and 21 GMT Jan. 1948-Feb. 1952, thereafter 03 GMT. Day observations were made at 12 GMT 1944-47, at 09 and 15 GMT Jan. 1948-March 1952, thereafter 15 GMT.

* Not significant at the 5 per cent level.

150 mb except in July when it is usually above 100 mb; the three levels 150, 100 and 60 mb were therefore chosen for the computations.

Stratospheric statistics are not given for Aden or the Pacific stations, as no significant correlations could be discovered from the data. Chiplonkar (1936) found an association between low pressure in the troposphere and high temperature at and above 16 km in autumn and winter over Agra (27°N, 78°E) but not at Poona (19°N, 74°E). There is a little evidence that temperatures in the lower stratosphere in the Tropics are more variable than in the upper troposphere and it may be that even in undisturbed weather (i.e. undisturbed in the lower troposphere) some relation may be found between tropopause and stratospheric temperatures. The greater range of stratospheric temperatures may merely be the result of their greater liability to error. Observations in the lower stratosphere are, as yet, too few for proper analysis.

TABLE 4. STRATOSPHERIC STATISTICS : LERWICK

	ρ_{H,P_T}	T	$\rho_{H,T}$	$\rho_{P_T,T}$	$\bar{H}\times10^{-1}$ (ft)	$\sigma_H\times10^{-1}$ (ft)	$\bar{P}_T$ (mb)	σ_{P_T} (mb)	$\bar{T}$ (°F)	σ_T (°F)	n
Jan.	− 0·77	T_{150}	− 0·58	0·74	2910	71	242	55	− 72·8	9·3	88
(night)		T_{100}	− 0·30	0·53					− 75·9	8·3	
		T_{60}	− 0·10*	0·32					− 81·2	9·1	
Feb.	− 0·75	T_{150}	− 0·42	0·62	2960	79	248	60	− 71·2	10·1	83
(night)		T_{100}	− 0·21*	0·37					− 74·4	9·7	
		T_{60}	− 0·01*	0·14*					− 78·8	12·3	
Mar.	− 0·73	T_{150}	− 0·56	0·67	2950	73	249	60	− 66·4	12·6	92
(day)		T_{100}	− 0·35	0·48					− 67·0	11·1	
		T_{60}	− 0·17*	0·31					− 66·6	12·7	
Apr.	− 0·71	T_{150}	− 0·42	0·65	2930	68	277	55	− 56·1	8·2	99
(day)		T_{100}	− 0·21	0·44					− 58·2	7·0	
		T_{60}	− 0·11*	0·29					− 58·0	7·7	
May	− 0·79	T_{150}	− 0·72	0·79	3000	52	249	47	− 54·9	7·8	113
(day)		T_{100}	− 0·71	0·76					− 54·0	5·7	
		T_{60}	− 0·43	0·45					− 50·1	6·3	
June	− 0·83	T_{150}	− 0·75	0·74	3040	60	253	46	− 50·8	6·8	118
(day)		T_{100}	− 0·67	0·72					− 50·7	5·1	
		T_{60}	− 0·20	0·27					− 46·5	4·7	
July	− 0·78	T_{150}	− 0·79	0·78	3050	57	237	37	− 51·5	8·4	93
(day)		T_{100}	− 0·72	0·64					− 49·5	5·9	
		T_{60}	− 0·47	0·35					− 44·7	5·0	
Aug.	− 0·83	T_{150}	− 0·79	0·78	3030	48	240	36	− 51·0	7·2	142
(day)		T_{100}	− 0·67	0·56					− 49·6	6·1	
		T_{60}	− 0·43	0·41					− 45·6	4·8	
Sept.	− 0·78	T_{150}	− 0·78	0·74	3010	68	237	52	− 58·1	9·4	148
(day)		T_{100}	− 0·70	0·67					− 57·5	8·0	
		T_{60}	− 0·40	0·39					− 55·0	7·1	
Oct.	− 0·83	T_{150}	− 0·83	0·77	2990	76	236	55	− 65·1	9·7	110
(day)		T_{100}	− 0·81	0·74					− 65·2	7·6	
		T_{60}	− 0·57	0·49					− 64·4	6·2	
Nov.	− 0·68	T_{150}	− 0·61	0·72	2940	68	249	55	− 70·8	9·1	103
(night)		T_{100}	− 0·54	0·65					− 75·0	7·0	
		T_{60}	− 0·24	0·38					− 81·1	7·1	
Dec.	− 0·74	T_{150}	− 0·55	0·75	2910	76	246	55	− 71·7	10·3	74
(night)		T_{100}	− 0·43	0·68					− 75·4	8·2	
		T_{60}	− 0·21*	0·44					− 80·8	9·0	

For explanation of period and times of observations, see Table 3. For symbols see Table 1.

* Not significant at the 5 per cent level.

TABLE 5. STRATOSPHERIC STATISTICS : ARCTIC BAY

	ρ_{H,P_T}	T	$\rho_{H,T}$	$\rho_{P_T,T}$	$\bar{H}\times10^{-1}$ (ft)	$\sigma_H\times10^{-1}$ (ft)	$\bar{P}_T$ (mb)	σ_{P_T} (mb)	$\bar{T}$ (°F)	σ_T (°F)	n
Jan.	− 0·54	T_{250}	− 0·39	0·73	2130	40	294	45	− 76	11·7	155
		T_{200}	− 0·36	0·75			302	49	− 71	11·1	96
		T_{150}	− 0·10*	0·56			297	52	− 71	11·1	44
Apr.	− 0·67	T_{250}	− 0·27*	0·68	2180	58	326	48	− 62	12·6	150
		T_{200}	− 0·25*	0·63			327	50	− 55	11·3	129
		T_{150}	− 0·28*	0·58			327	49	− 52	9·1	113
July	− 0·81	T_{250}	− 0·77	0·77	2330	41	271	44	− 56	10·7	164
		T_{200}	− 0·78	0·77			271	44	− 49	9·4	162
		T_{150}	− 0·69	0·66			272	44	− 46	6·2	151
Oct.	− 0·55	T_{250}	− 0·56	0·64	2190	34	309	37	− 58	8·2	88
		T_{200}	− 0·44	0·54			310	36	− 55	5·9	75
		T_{150}	− 0·25*	0·41			305	35	− 53	5·4	56

Period : January 1947-51; other months 1948-50. H = height of 400-mb level. Symbols as for Table 1.

* Not significant at 5 per cent level (effective number of observations may be less than shown above by reason of persistence).

TABLE 6. STRATOSPHERIC STATISTICS : MALTA

	ρ_{H,P_T}	T	$\rho_{H,T}$	$\rho_{P_T,T}$	$\bar{H}\times10^{-1}$ (ft)	$\sigma_H\times10^{-1}$ (ft)	$\bar{P}_T$ (mb)	σ_{P_T} (mb)	$\bar{T}$ (°F)	σ_T (°F)	n
Jan.	− 0·72	T_{150}	− 0·64	0·68	3010	42	234	45	− 68	7·6	189
		T_{100}	− 0·55	0·47	3010	41	233	45	− 75	5·9	162
		T_{60}	− 0·34	0·37	3020	37	222	47	− 78	6·1	58
Apr.	− 0·78	T_{150}	− 0·61	0·71	3050	38	215	39	− 69	7·6	170
		T_{100}	− 0·41	0·42			215	38	− 74	5·5	142
		T_{60}	− 0·11*	0·19*			218	41	− 72	5·5	43
July	− 0·16	T_{150}	-	− 0·36	3170	23	99	19	− 72	4·5	142
		T_{100}	− 0·27	0·11*			98	19	− 85	4·8	141
		T_{60}	− 0·30	0·34			96	19	− 80	7·2	87
Oct.	− 0·35	T_{150}	− 0·38	0·62	3100	31	181	32	− 76	6·8	202
		T_{100}	− 0·27*	0·38	3100	31	180	30	− 78	6·3	177
		T_{60}	− 0·07*	− 0·12*	3110	26	179	21	− 76	6·3	79

Period : 1948-51. H = height of 300-mb level. Symbols as for Table 1.

* Not significant at 5 per cent level (effective number of observations may be less than shown above by reason of persistence).

TABLE 7. UNSELECTED STATISTICS : LARKHILL AND LERWICK

	$\bar{H}\times10^{-1}$ (ft)	$\sigma_H\times10^{-1}$ (ft)	$\bar{P}_T$ (mb)	σ_{P_T} (mb)	$\bar{T}_{150}$ (°F)	$\sigma_{T_{150}}$ (°F)	$\bar{H}\times10^{-1}$ (ft)	$\sigma_H\times10^{-1}$ (ft)	$\bar{P}_T$ (mb)	σ_{P_T} (mb)	$\bar{T}_{150}$ (°F)	$\sigma_{T_{150}}$ (°F)
	Larkhill						Lerwick					
Jan.	2959	77	236	54	− 71·3	10·6	2896	76	254	57	− 71·3	9·5
Feb.	2955	64	240	53	− 69·9	11·5	2900	75	256	54	− 68·9	11·1
Mar.	2983	49	227	43	− 69·6	9·7	2930	68	247	50	− 68·3	10·8
Apr.	2989	67	242	56	− 64·0	8·8	2925	65	276	57	− 57·6	8·6
May	3014	52	241	43	− 62·4	7·2	2989	73	251	47	− 57·5	7·4
June	3050	58	231	53	− 60·4	7·7	3002	62	259	48	− 51·4	7·9
July	3079	44	225	39	− 59·7	7·9	3031	51	248	37	− 51·1	7·0
Aug.	3071	48	228	36	− 60·3	8·6	3028	45	240	32	− 53·7	7·7
Sept.	3062	48	220	41	− 67·1	9·2	3001	58	244	49	− 58·9	8·0
Oct.	3036	66	223	50	− 71·3	8·8	2991	71	235	53	− 67·0	9·7
Nov.	2986	70	234	57	− 72·8	10·3	2930	66	256	55	− 68·6	9·1
Dec.	2962	66	239	59	− 71·3	9·0	2905	80	258	63	− 70·0	10·8

These statistics were evaluated from the following data :

$\bar{H}$, $\bar{P}_T$, $\bar{T}_{150}$ – four observations, daily Jan. 1944-Mar. 1952; two observations, daily April 1952-1953.

σ_H, σ_{P_T}, $\sigma_{T_{150}}$ – four observations daily, 1946-50 except T_{150} for January, April, July and October, which were evaluated from all observations made in 1942-51.

Symbols as for Table 1.

5. Significance and accuracy of the statistics

(*a*) *Diurnal variation of temperature*

The regular diurnal variation of temperature observed in the stratosphere may be expected to reduce the correlation coefficients. This effect will only apply to statistics for Arctic Bay and Malta. The reduction will only be appreciable when the variation is such that the square of the ratio of its standard deviation to the standard deviation of temperature is comparable with unity. At Malta the mean diurnal variation of temperature at 60 mb, i.e., the difference between the means of the 03 and 15 GMT observations, is approximately 4°F in January, 6°F in April, 7°F in July and 5°F in October. These values were obtained from the observations made in the period 1948-1954. Diurnal variation is less at 100 mb and lower levels. For the purposes of the present argument it does not matter whether the diurnal variation is real or is caused by radiation errors of the instrument. The diurnal variation at Malta at 60 mb is thus equivalent to a variation about the mean having a standard deviation of 2°F in January, 3°F in April, 3·5°F in July and 2·5°F in October. In April, therefore, the standard deviation appropriate to the diurnal variation is about 0·55 of the standard deviation of the variation of temperature at 60 mb from all sources. For the other months the ratios are smaller. The correlation coefficients for T_{60} for April for Malta (Table 6) are therefore probably a little too low because of the effects of diurnal variation and a simple calculation shows that the corrections are of the order of 15 per cent and thus unimportant. Corrections should be smaller still for other months and for lower levels.

Diurnal variation of temperature in the upper air at Arctic Bay would be expected to be small since observations are made at about 21 and 09 LT (03 and 15 GMT). This was found to be the case. Differences between monthly mean-temperatures at 150 mb for the two hours for January, April, July and October for the three years investigated were at most 2·7°F and showed the 03 GMT means less than the 15 GMT means in January and April, but the reverse in July and October, thus indicating no systematic diurnal variation. Radiation effects would be expected to be most noticeable in April and October when there is night and day at Arctic Bay. For other levels below 150 mb, except near the ground, diurnal variation of temperature should also be small.

(*b*) *Casual errors of observation*

Casual instrumental errors of the British-type radiosonde have a standard deviation of 2°F in the stratosphere and considerably less in the troposphere (Harrison 1948) so that their effect on the correlation coefficients will be even less than that of the diurnal variation.

Casual observational errors of the U.S.A.-type radiosonde used at Arctic Bay are not known, but may be assumed to be of the same order of magnitude as the errors of the British radiosonde.

(*c*) *Persistence*

Temperature observations at a fixed level above a station will show some persistence from one observation to the next. Carruthers (1949) quotes a 'persistence length' of three days for daily temperature readings at a fixed hour in a conventional screen near the earth's surface. Lewis and McIntosh (1952) find five as the 'equivalent number of repetitions' for the departures of the Edinburgh daily mean temperature for 1950 from the corresponding 50-yr average.

To discover the amount of persistence in a series of upper-air temperatures Lewis and McIntosh's (1952) procedure was applied to T_{500} and T_{150} for Lerwick for January and July 1950 and for Malta for January and July 1951; also to T_{500} for January 1948 and July 1950 and to T_{200} for July 1949 for Arctic Bay. There were insufficient observations at 200 mb at Arctic Bay in any January to make an estimate of persistence. The results are given in Table 8, the persistence being calculated from the formula (2) of Lewis and McIntosh's paper for the ' equivalent number of repetitions.'

TABLE 8. PERSISTENCE (IN HOURS) OF UPPER-AIR TEMPERATURE

	January		July	
	500 mb	150 mb	500 mb	150 mb
Lerwick	36	40	48	27
(1950)	Q	Q	A	Q
Malta	66	42	60	72
(1951)	A	A	A	A
	500 mb	200 mb	500 mb	200 mb
Arctic Bay	66	–	66	60
	A		A	Q

A = standard deviation of temperature near average.
Q = standard deviation of temperature more than 1°F below average.

The figures can only give a rough estimate of the persistence, as presumably this will depend to some extent on the character of the weather in the particular month chosen for scrutiny. The variability of temperature in those particular months is some indication of their character and the letters A (average) and Q (quiet) are used for this purpose in the table, A signifying that the standard deviation of temperature is roughly the same as, and Q that it is more than 1°F less than, the appropriate long-term standard deviation.

It would seem that for Arctic Bay and Malta (Tables 5 and 6) the number of independent observations is only 30-40. For the higher levels, where the numbers of observations are much reduced, the series will have less coherence than a full series; it is impossible to estimate the correct number of independent observations without investigating where observations are missing in the series, but assuming a fair measure of scatter of the observations throughout the period the number of independent observations at Arctic Bay may be assumed to be 20-25 at 150 mb in January and about 30 in July. Similarly, the number of independent observations at 60 mb at Malta is about 30 in April and rather more in other months.

The observations used in preparing Tables 3 and 4 were restricted to certain hours of the day and are few in number compared with the number of possible observations had every sounding reached 60 mb. Assuming that the observations were evenly distributed in time the interval between each observation was more than two days. This is not a valid assumption but even so it is likely that the set of observations analysed in Tables 3 and 4 is little affected by persistence.

6. Discussion

(*a*) *Larkhill*

Dines's statement that low tropopause is, in general, associated with warm lower stratosphere, and high tropopause with cold, is confirmed by the correlations in Table 3. The stratospheric coefficients are seen to be lowest in winter and highest in October. The seasonal variations and the decrease of the coefficients with height in the stratosphere are discussed in Sections 7 and 8.

Comparison of the unselected means in Table 7 with the means of the data used in the correlations, shows that there is a tendency to higher tropopauses (i.e., lower P_T), larger values of H_{300}, and lower temperatures at 150 mb, in the observations which reached 60 mb than in the unselected data. The reason for this is uncertain but may be because of the tendency for quieter weather in the troposphere to be associated with higher tropopauses, etc.; in unsettled weather, especially when there are strong winds near the earth's surface, the radiosonde balloon will be buffeted in the early stages of its flight and will be more likely to burst prematurely. The slight bias in the observations reaching 60 mb cannot have an appreciable effect on the magnitude of the correlation coefficients set out in Table 3.

(*b*) *Lerwick*

The stratospheric coefficients for Lerwick (Table 4) are roughly similar to those evaluated for Larkhill (Table 3) or are slightly lower. The differences between the mean values of the data used in the correlations and the means of the unselected data in Table 7 are smaller, in general, than for Larkhill, although again the bias in the selected data for the stratosphere is towards higher H_{300} and lower P_T and T_{150}.

(*c*) *Arctic Bay*

At Arctic Bay the levels 250, 200 and 150 mb are in the stratosphere on most occasions. Of the observations used in preparing the statistics in Table 5, the following numbers had tropopauses above the level of 250 mb :

January	April	July	October
19	3	53	3

The correlation coefficients for July for T_{250} are probably not truly representative of the lower stratosphere, but for the other three months the level of 250 mb may be considered stratospheric.

In July and, to a lesser extent, in October, the stratospheric coefficients are roughly similar to those for the British Isles. In January and April, however, temperatures in the lower stratosphere are not closely associated with H_{400} (height of 400 mb level) although they are correlated with P_T to much the same extent as at Lerwick and Larkhill. The relation between H_{400} and P_T is, in general, less than the corresponding relation between H_{300} and P_T for Lerwick and Larkhill, except in July.

At Arctic Bay in winter and spring there is seldom, if ever, a sequence of weather systems with changing air masses. The upper air (500 mb) in these months is dominated by a ' low,' which is usually west of Greenland and south-east of Arctic Bay, but which oscillates about this position and can have secondary ' lows ' associated with it. In summer and also, to a lesser extent, in autumn, frontal depressions affect Arctic Bay, though much less frequently than they do the British Isles; the main polar-front disturbances are much further to the south. The uniform radiative properties of the vast area of snow or ice in the Arctic, especially in winter and spring, are probably responsible for this uniformity of air mass. It is not surprising, therefore, that the model of upper-air temperature relations which applies to western Europe does not apply so well to the arctic north of Canada. Reasonable deductions from the correlation coefficients of Table 5 are :

(i) that temperatures in the lower stratosphere are influenced by vertical motion to the same extent as over the British Isles (P_T and T_{250}, T_{200}, T_{150}).

(ii) vertical motion at the level of the tropopause is positively correlated with pressure at a fixed level near the top of the troposphere (from the H_{400}, P_T correlation). The association is not strong but is probably similar to that over the British Isles for the following reasons. Over Britain, advection plays some part in the association of H_{300} and P_T; Sawyer (1954) estimates it as similar in magnitude to the effect of vertical motion. The correlation between H_{300} and subsidence at tropopause level would thus be approximately $2^{-\frac{1}{2}}$ times the H_{300}, P_T correlation (i.e. about 0·5 or 0·6). Over Arctic Bay advection is probably much less important, except in July; the H_{400}, P_T correlations of about 0·6 in January, April and October probably arise through the effect of subsidence, the July coefficient (0·81) being partly caused by advection also.

(*d*) *Malta*

In winter and spring the central Mediterranean is affected by travelling disturbances common to temperate latitudes. The correlations in Tables 1 and 6 reflect this, being only a little less than for Larkhill for the same months. In summer, however, Malta's weather is influenced by the sub-tropical high-pressure belt, the tropopause is high (about 100 mb) and the temperate latitude correlations are not found. In October the summer weather is breaking and lower tropopauses are found again on many days (Meteorological Office 1954); there is some correlation between P_T and stratospheric temperatures but little relation between the other variables.

As stated previously, the tropopause is often double at Malta and as a general rule the lower tropopause was selected for the present analysis. In January and April there is a low-level tropopause (i.e., roughly in the range 150-300 mb) on almost every occasion; on perhaps one-third of occasions there is an upper tropopause (60-120 mb) as well. The analysis took no account of these upper tropopauses, so that the correlations involving T_{150} and T_{100} are not entirely representative of true stratospheric conditions as up to one-third of the observations at these two levels were below the upper tropopause. In July a lower tropopause occurs on a few days but usually there is a single high tropopause. In October a few days occur with a single high tropopause; on others there may be a single low tropopause or a double tropopause. If two tropopauses were reported the lower was accepted for the present analysis, so that in the October data there is a mixture of a few high-level (tropical) with the predominating low-level (temperate) tropopauses. This, however, does not fully explain the low correlations between P_T and tropospheric parameters in October; the correlations between P_T and T_{500} and P_T and T_{100} were evaluated for the years 1950 and 1951 separately, using only those occasions in October when a low tropopause (i.e., P_T greater than or equal to 150 mb) was found. The correlation coefficients between P_T and T_{500} were $-0{\cdot}66$ (48 observations) and $-0{\cdot}36$ (50 observations) in 1950 and 1951 respectively; coefficients for P_T and T_{100} were $+0{\cdot}42$ (43 observations) and $+0{\cdot}40$ (41 observations).

7. Variation of the correlations with height in the stratosphere

It is seen from Tables 3, 4, 5 and 6 that when there is a relation between temperature in the lower stratosphere and the pressure at the tropopause (P_T) or the height of a fixed pressure surface in the upper troposphere (H_{300} or H_{400}), this relation decreases with increasing height. The decrease in the correlation coefficients upwards shown in

Table 6 may be caused in part by the greater diurnal variation of temperature at the greater heights (see Section 5 (*a*)), but the tendency to decrease is almost certainly real. This adverse effect of radiative errors will be very small in Tables 3 and 4, which were computed from observations in similar radiative conditions.

The square of the correlation coefficient between two variables is a measure of the dependence of one variable on the other. Table 9 gives the square of the ratio of the coefficients for T_{60} and T_{150} for Larkhill, Lerwick and Malta and for T_{150} and T_{250} for Arctic Bay; values for Malta for July are omitted as of no significance. The figures in this table are an indication of the reduction with height in the relation between stratospheric temperatures and, respectively, P_T and the height of a constant pressure surface in the upper troposphere (H_{300} or H_{400}).

TABLE 9. RATIO OF THE SQUARES OF CORRELATION COEFFICIENTS FOR TWO LEVELS IN THE STRATOSPHERE

		J	F	M	A	M	J	J	A	S	O	N	D	Mean†
Larkhill	A	0·18	0·04*	0·25	0·10	0·28	0·38	0·36	0·40	0·33	0·46	0·09*	0·02*	0·21
	B	0·38	0·50	0·50	0·49	0·42	0·51	0·60	0·55	0·44	0·64	0·43	0·37	0·49
Lerwick	A	0·04*	0·00*	0·09*	0·02	0·36	0·07	0·35	0·30	0·26	0·47	0·15	0·15*	0·15
	B	0·19	0·05*	0·21	0·18	0·32	0·13	0·20	0·28	0·28	0·40	0·28	0·34	0·23
Malta	A	0·28			0·03*						0·03*			0·09
	B	0·30			0·07*						0‡			0·07
Arctic Bay	C	0·07*			1·0*			0·80			0·20*			0·42
	D	0·59			0·73			0·73			0·41			0·61

A = ratio of square of correlation coefficients between H_{300} and respectively T_{60} and T_{150}.
B = ratio of square of correlation coefficients between P_T and respectively T_{60} and T_{150}.
C = ratio of square of correlation coefficients between H_{400} and respectively T_{150} and T_{250}.
D = ratio of square of correlation coefficients between P_T and respectively T_{150} and T_{250}.

* One or more of the correlation coefficients not significant at the 5 per cent level.

† The values here are the squares of the mean ratios.

‡ The coefficient for T_{60} is small and has opposite sign to that for T_{100} and T_{150} and is taken as zero.

It is clear from Table 9 that the relation between temperature in the lower stratosphere and H_{300} and P_T respectively is reduced by a factor of about two or more between 150 and 60 mb at Larkhill and Lerwick. The decrease is greater at Lerwick than at Larkhill and is not so large for P_T as it is for H_{300}. The decrease is more marked in winter than in late summer. At Malta the decrease with height in these relations is very marked.

The corresponding figures for Arctic Bay show only a small reduction with height in the relation between stratospheric temperatures and H_{400} and P_T respectively, but observations were not available for levels above 150 mb and it is probable that the relationship decreases above this level as it does at Lerwick, Larkhill and Malta.

8. SEASONAL VARIATIONS IN THE CORRELATIONS

Tables 3 (Larkhill) and 4 (Lerwick) provide the data for a study of the seasonal variation in the stratospheric relationships. The coefficients vary somewhat erratically, as might perhaps be expected from random samples. Using Fisher's Z' transformation (see, e.g., Brooks and Carruthers 1953), it is easily shown that any of the sets of 12 monthly

correlation coefficients for one station (e.g., between H_{300} and T_{150}) is very unlikely (probability less than 1 in 100) to have arisen from the same population, i.e., the differences in the correlation coefficients are significant and indicate real changes from month to month.

TABLE 10. CONFIDENCE LIMITS OF CORRELATION COEFFICIENTS COMPUTED FROM 103 PAIRS OF VALUES

Computed correlation coefficient	0·3	0·5	0·7	0·9
95 per cent confidence limits	0·11 0·47	0·34 0·63	0·58 0·79	0·85 0·93

However, some of the variation in the coefficients is likely to be the result of ordinary sampling errors. It can be shown (again by the use of Fisher's Z' transformation) that for the number of observations (about 100) used in computing each coefficient, a calculated correlation coefficient of a value shown in Table 10 can be interpreted as indicating that the true value of the correlation coefficient will lie between the limits shown on all but 5 per cent of occasions. With these figures in mind it is clear that only the broad pattern of seasonal changes in the correlation coefficients can be inferred from the values in Tables 3 and 4. In Fig. 1 the correlation coefficients for T_{150} and T_{60} with H_{300} for Larkhill are plotted for each month and a smooth curve is drawn through these points. Fig. 2 is a similar diagram for T_{150} and T_{60} with P_T for the same station, and Figs. 3 and 4 are corresponding diagrams for Lerwick. These curves are necessarily somewhat subjective, but the following deductions may be made from them :

(i) The H_{300}, T_{150} relation is comparatively large from May onwards, reaching a maximum in October at both stations, thereafter falling steeply to a minimum in January-February. The range is significantly greater at Lerwick than at Larkhill.

(ii) The H_{300}, T_{60} relation is similar to (i) but the annual range is greater.

(iii) At both Larkhill and Lerwick the P_T, T_{150} relation has a flat maximum in midsummer and a minimum in January-February. The annual range is slightly greater at Lerwick than at Larkhill.

(iv) The P_T, T_{60} relation is similar to (iii) but the maximum at Lerwick is displaced to October and there is a similar tendency at Larkhill (September value makes the latter conclusion tentative). The annual range is larger than for P_T, T_{150}.

With the exception of H_{300}, T_{150} for Larkhill, the plotted points in Figs. 1-4 show a tendency for a reduction of the relations in April. This may well be real, though this tendency has not been shown in the smooth curves. The figures in Table 7 show that temperature in the stratosphere is increasing most rapidly in April and this will tend to reduce the correlation coefficients. Similarly, the smaller correlation coefficients for T_{150} in September than in October may be a result of temperatures falling more rapidly in the lower stratosphere in September.

The annual variation of the relation between temperatures in the lower stratosphere and the tropospheric parameter H_{300} and the partly tropospheric parameter P_T is no doubt associated with the large seasonal changes in the temperature structure of the stratosphere. For most of the year the mean temperature in the lower stratosphere increases northwards over the northern hemisphere. In winter, however, the arctic stratosphere is relatively cold so that the mean temperature distribution in January at 150 mb, for example, shows low temperature over the Pole, a belt of higher temperatures in middle latitudes (axis about 55°N over Britain), and cold again over the Tropics (authority –

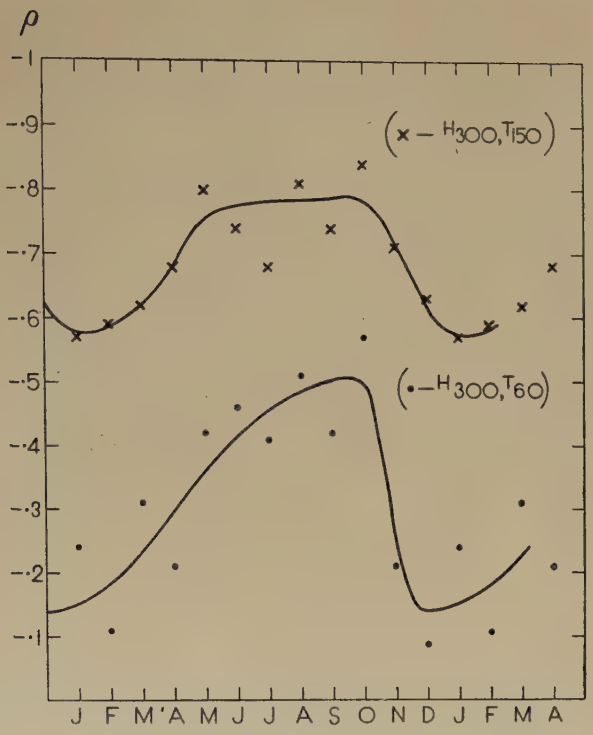

Figure 1. H_{300} with T_{150} and T_{60} – Larkhill.

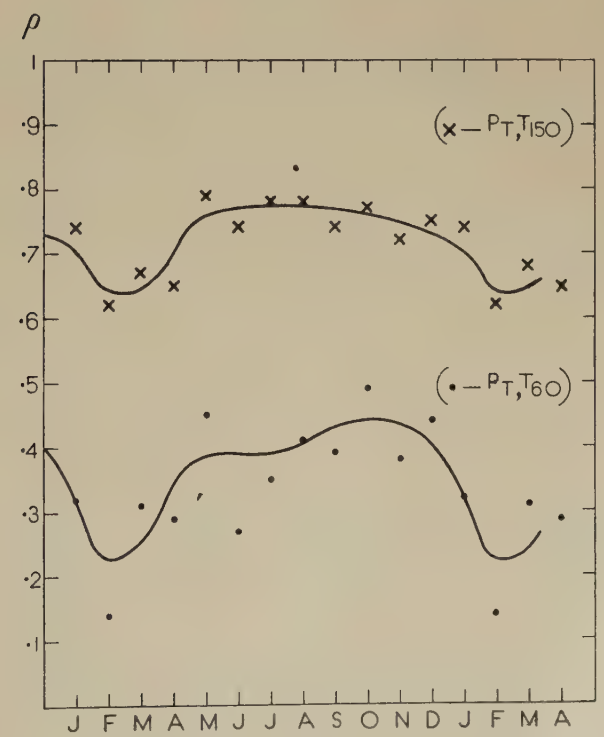

Figure 2. P_T with T_{150} and T_{60} – Larkhill.

Annual variation of stratospheric correlation coefficients.

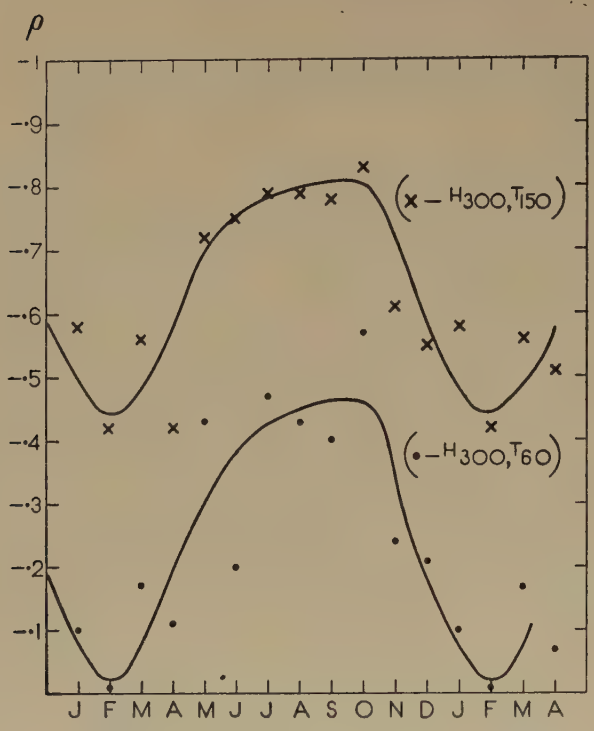

Figure 3. H_{300} with T_{150} and T_{60} – Lerwick.

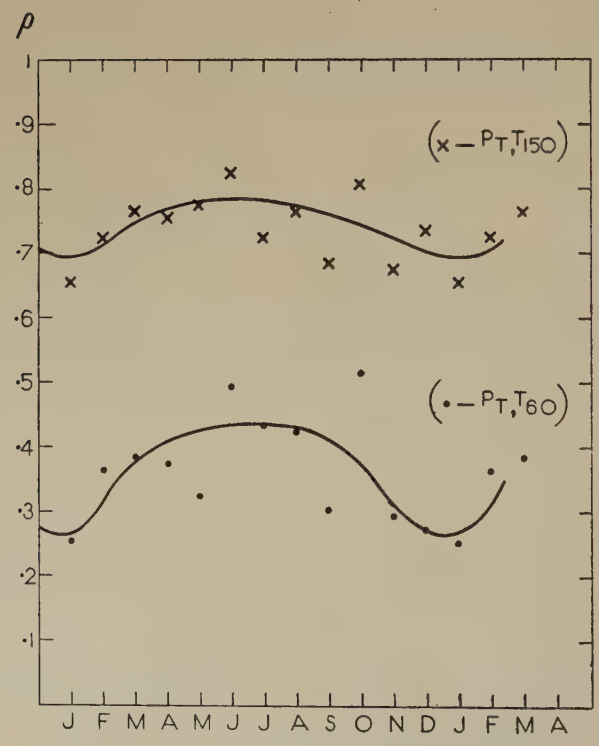

Figure 4. P_T with T_{150} and T_{60} – Lerwick.

Annual variation of stratospheric correlation coefficients.

unpublished charts in the Meteorological Office). Deviations from the mean distribution occur, of course, so that on some occasions Britain may be in or to the south of the warm stratospheric belt and on others, following an excursion of arctic air southwards, it will be within or on the edge of the Arctic cold stratosphere. This is confirmed by the winds which on some occasions decrease with height in the stratosphere and on others increase with height when on the edge of the cold arctic stratosphere (Austin and Bannon 1952). The effect is most noticeable at Lerwick. The temperature often decreases slowly with height in the stratosphere over Britain in winter, as has been remarked by Miles (1952), though on other occasions the normal increase with height is noted. Riehl (1950) has shown that over arctic Canada in winter the lower stratosphere may be colder or warmer than that to the south, although the general weather pattern at the surface is similar.

In winter the stratospheric correlation coefficients for Lerwick are less than those for Larkhill, in keeping with the greater variability in the type of temperature distribution in the lower stratosphere over the north of the British Isles than in the south. The north is more often affected by cold arctic currents with associated arctic-type stratospheres. The greater decrease with height at Lerwick than at Larkhill of the stratospheric coefficients in winter (see Section 7) also agrees with this interpretation.

9. Conclusions

This statistical investigation has confirmed, for the British Isles, the relations between upper-air temperatures and the level of the tropopause and the pressure in the upper troposphere respectively which Dines deduced quantitatively for the troposphere and qualitatively for the lower stratosphere from observations made over Europe. At Arctic Bay, however, relations of the Dines type are much weaker in winter and spring, though they are evident in summer when frontal depressions affect the area and, to a limited extent, in autumn also. On the other hand, at Malta such relations only hold in winter and spring when temperate-latitude weather systems affect the Mediterranean and not at all in summer when stable sub-tropical weather prevails; in autumn, though the weather is reverting towards the type of more temperate latitudes the relations are still weak. There appear to be no relations of the Dines type in the Tropics, though the observations in the lower stratosphere are insufficient to show this with certainty.

A reasonable generalization from these results, and one which is not surprising, is that the Dines relationships hold only in those regions affected by the travelling depressions and anticyclones of temperate latitudes.

A rapid decrease of the correlation coefficients with height in the lower stratosphere has been found, except at Arctic Bay where observations are not available in any number at high altitudes. The numerical values obtained indicate that the influence at the 60-mb level of the travelling depressions and anticyclones of middle latitudes is less than half that at 150 mb.

The marked winter minima in the stratospheric correlation coefficients for Larkhill and Lerwick are probably the result of a cold arctic-type stratosphere which is brought over Britain, especially the northern parts, from time to time during the winter.

Acknowledgments

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551.558.21

Airflow over an isolated hill

By R. S. SCORER
Imperial College, London

(Manuscript received, 26 September 1955)

SUMMARY

Perturbation theory is used to compute the vertical displacement of a uniform airstream passing over a solitary hill : first for a circular hill, then for a hill of oval section lying across the wind, obliquely to it and along it. The diagrams show contours of the vertical displacement at four different heights above the hill. It is significant that a small but steep-sided oval hill produces almost as much displacement as a larger circular, less-steep hill. When inclined obliquely to the airstream the pattern is very asymmetrical, the displacement being greater over the end of the hill that is farther down wind. A hill lying along the wind produces a very small effect.

1. INTRODUCTION

The notation used is the same as in two previous papers of 1953 and 1954, here referred to as II and III respectively. Additional notation is explained as required.

If the flow can be calculated for a two-dimensional ridge inclined obliquely to an airstream then the flow over an isolated hill may be obtained by superposition (integration) of the flow over a set of ridges inclined at different angles to the flow. The shape of the isolated hill depends upon the relative height of the component ridges that are superimposed.

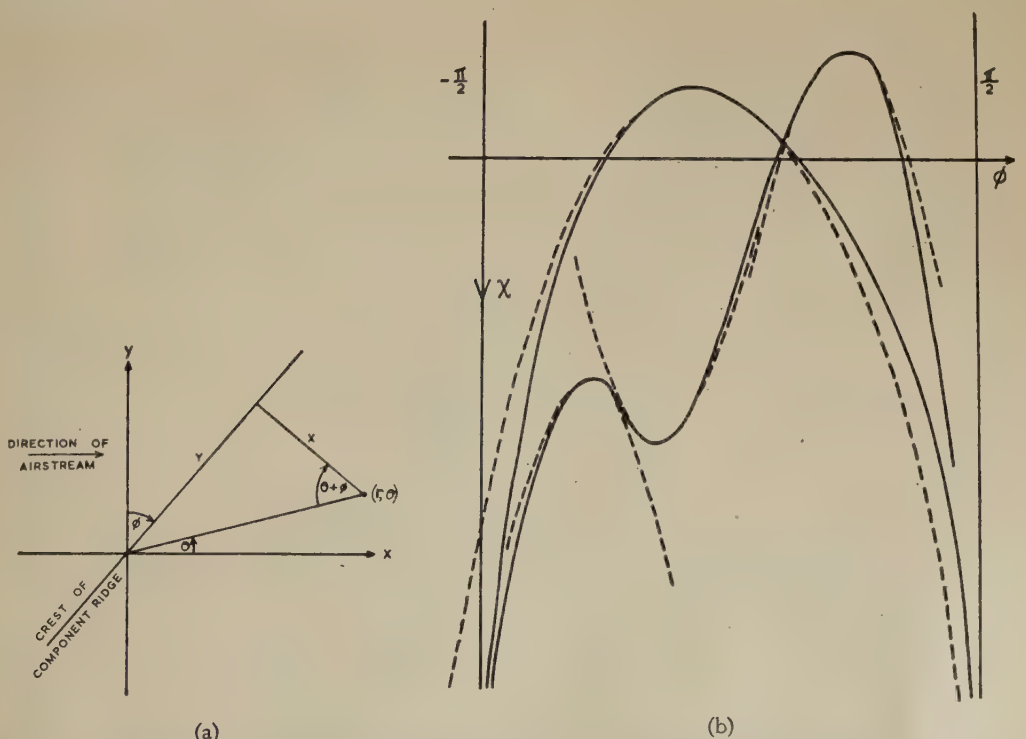
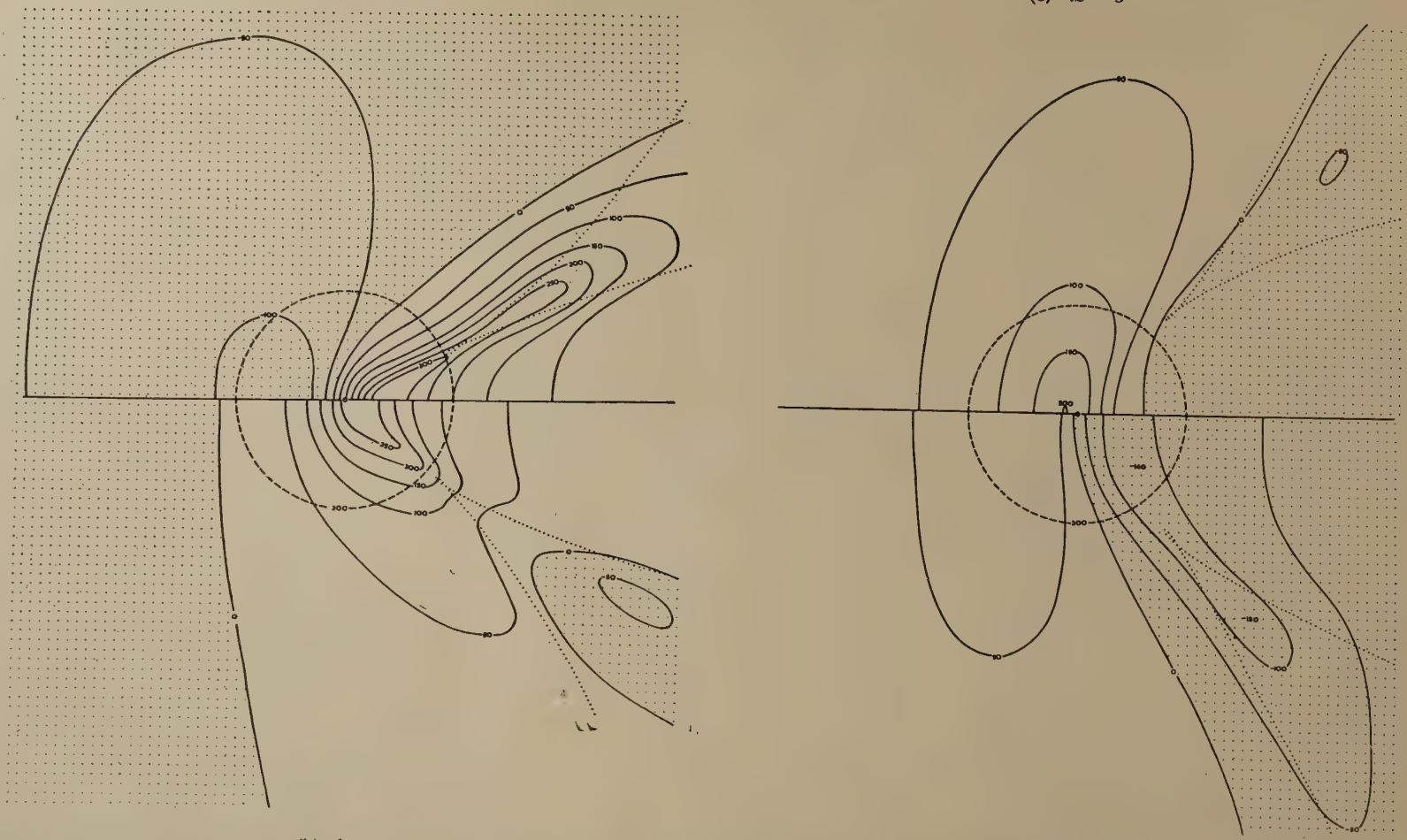


Figure 1. (a) System of coordinates in a horizontal plane. (b) Possible shapes of the function χ (full curves) and the parabolas (dashed curves) substituted in the method of stationary phase.

(a) $lz = 1$

(c) $lz = 3$

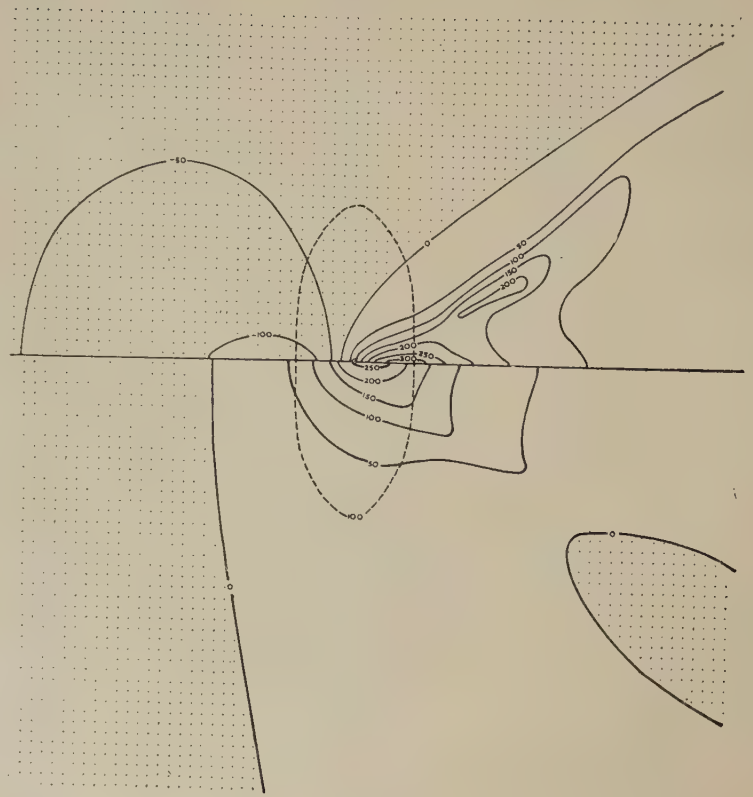


(b) $lz = 2$

(d) $lz = 4$

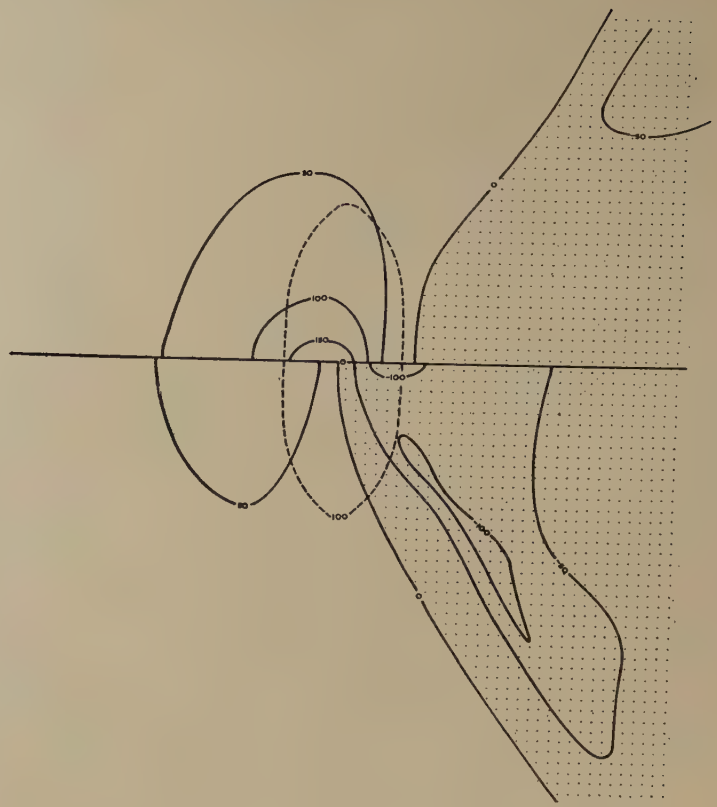
Figure 3. Vertical displacement over a circular hill. (For explanation see p. 80).

(a) $lz = 1$



(b) $lz = 2$

(c) $lz = 3$



(d) $lz = 4$

Figure 4. Vertical displacement over an oval hill lying across the wind. $\Phi = 1$ when $\phi = 90°$. (For explanation see p. 80).

If the component ridge is inclined at angle ϕ (see Fig. 1 (a)) to the normal to a uniform airstream and is of amplitude Φ then it was shown in III, Eqs. (14) and (15), that the ground height ζ_1, and the vertical displacement ζ take the form

$$\zeta_1 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \Phi \frac{a^2 b}{b^2 + X^2} d\phi \quad . \quad . \quad . \quad . \quad (1.1)$$

$$\zeta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \Phi a^2 (b^2 + X^2)^{-\frac{1}{2}} e^{ix} d\phi \text{ (real part)} \quad . \quad (1.2)$$

where

$$\left.\begin{aligned} X &= r \cos(\theta + \phi), \quad r \text{ and } \theta \text{ being horizontal} \\ &\qquad\qquad\qquad\qquad \text{polar coordinates} \\ x &= \tan^{-1} X/b - lz \sec \phi \end{aligned}\right\} \quad . \quad (1.3)$$

If Φ is independent of ϕ the hill is circular. In a second case calculated we arbitrarily chose

$$\Phi = \cos\left\{\left(\frac{2\phi}{\pi}\right)^{\frac{1}{3}} \frac{\pi}{2}\right\} \quad . \quad . \quad . \quad . \quad (1.4)$$

which, on evaluating (1.1) gave the hill whose contours are represented in Figs. 2 (a & b). In a third case we wrote $(\phi - 40°)$ for ϕ in Eq. (1.4) to obtain the same hill rotated through 40°, so that it was lying obliquely to the airstream, and finally, by a similar method, we calculated the displacement when the wind was along the long axis of the hill.

These hill shapes were found by a form of Weddle's rule giving a numerical value of the integral (1.1). Values of the integrand at 10° intervals of ϕ were used.

2. The calculation of the vertical displacement

x behaves, as a function of ϕ, like one of the curves in Fig. 1 (b), over the range of integration $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Since x tends to infinity towards the ends of the range, e^{ix} is highly oscillatory and no contribution to the integral is made there. It is appropriate therefore to use the principle of stationary phase to evaluate (1.2), provided that Φ and $(b^2 + X^2)^{-\frac{1}{2}}$ are slowly varying in comparison with cos x in the middle of the range, which is the case.

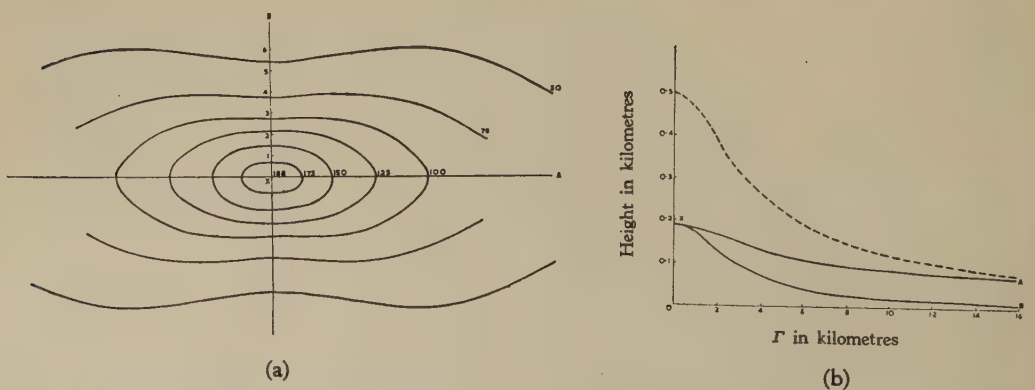


Figure 2. (a) Contours of the oval hill obtained by using the function Φ specified in Eq. (14.). (b) Vertical section of the circular hill (dashed curve) and two sections of the oval hill (full curves).

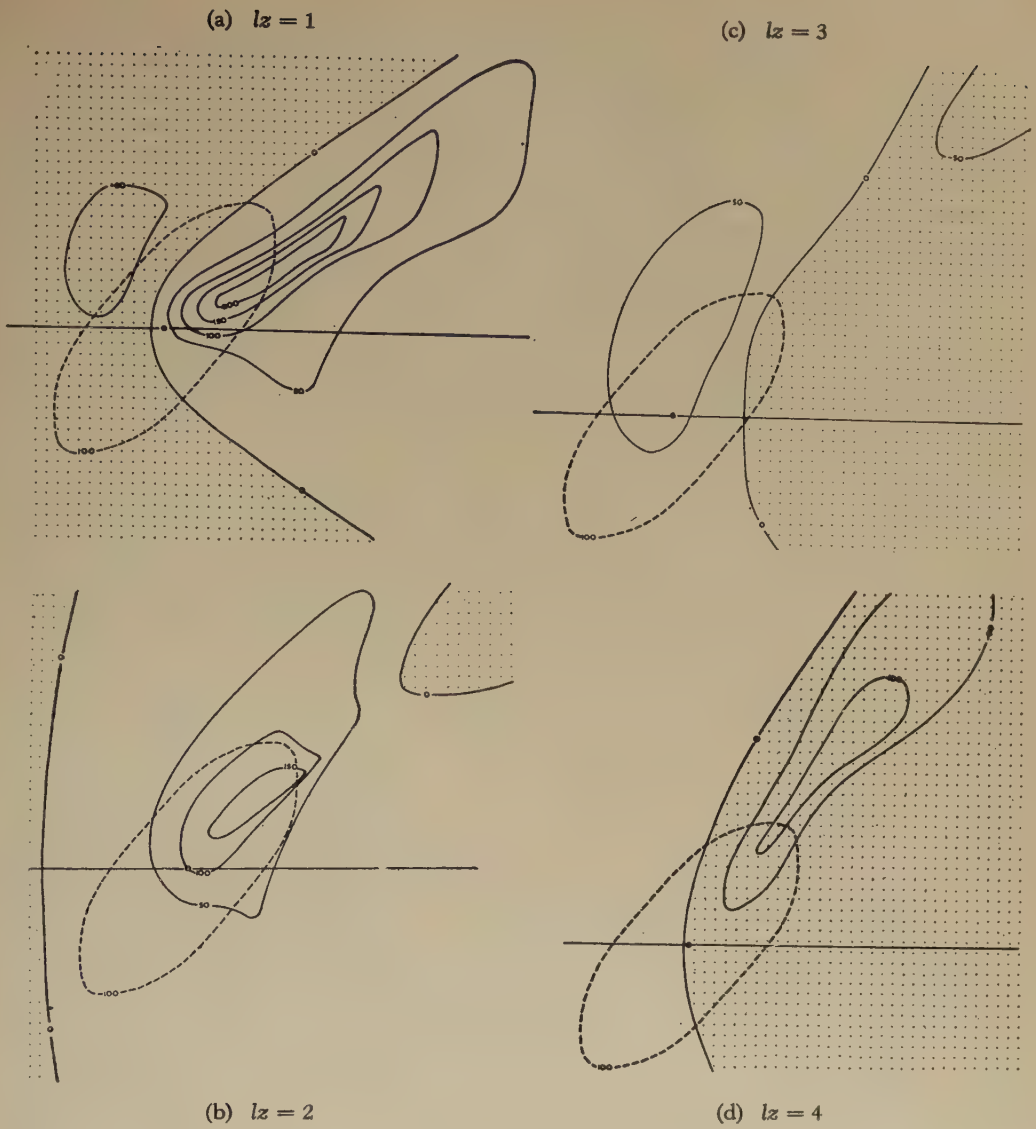


Figure 5. Vertical displacement over an oval hill inclined at 40° to the normal to the wind. $\Phi = 1$ when $\phi = 40°$. (For explanation see p. 80).

The method is equivalent to replacing the curve of x by a parabola (or parabolas, see Fig. 1 (b)) which touches the actual curve where ϕ satisfies

$$\frac{\partial x}{\partial \phi} = 0 \quad . \quad . \quad . \quad . \quad . \quad (2.1)$$

and replacing $\Phi\,(b^2 + X^2)^{-\frac{1}{2}}$ by its value at this particular value of ϕ over the whole range of integration. Only close to where the tangent to the curve (or parabola) is nearly horizontal is there an appreciable contribution to the integral. The method gives a good approximation to the integral except close to the values of ϕ at which $\partial^2 x/\partial\phi^2 = 0$, which occur only for particular values of r, θ, z.

The value of the integral is required at points of space r, θ, z. It is most convenient to evaluate it for each z for chosen values of X, ϕ and then to find the corresponding

values of r, θ afterwards. Since ϕ satisfies Eq. (2.1), viz.,

$$-bY/(b^2 + X^2) - lz \sec\phi \tan\phi = 0 \quad . \quad . \quad (2.2)$$

where $$Y = r \sin(\theta + \phi) \quad \text{(see Fig. 1 (a))} \quad . \quad . \quad (2.3)$$

we have $$Y = -l\frac{z}{b}(b^2 + X^2) \sec\phi \tan\phi \quad . \quad . \quad . \quad (2.4)$$

When X, Y and ϕ are known, r, θ or x, y can easily be obtained graphically; in this case simply by rotating, through angle ϕ, tracing paper superimposed on graph paper.

Close to the line along which $\partial^2 x/\partial\phi^2 = 0$ the integral was not evaluated. On one side of this line there was only one value of ϕ satisfying Eq. (2.1), on the other side there were three values corresponding to the three parabolas in Fig. 1 (b), and these were added together. The region close to the line was filled in by drawing the contours across from one side to the other smoothly.

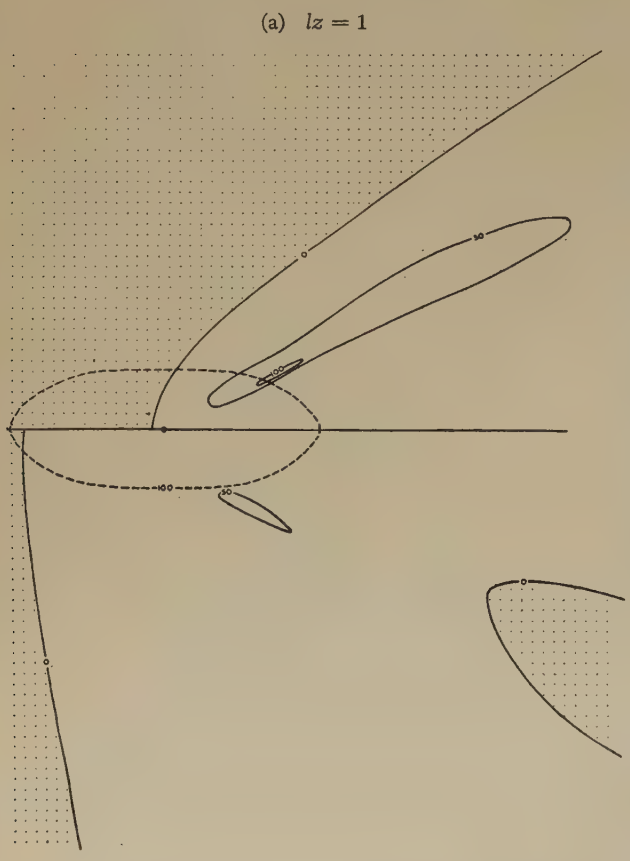


Figure 6. Vertical displacement over an oval hill lying along the wind. $\Phi = 1$ when $\phi = 90°$; (c), (d) are not drawn because when $lz = 3$, 4 the effect is negligible.

Figures 3 to 6. Vertical displacements of uniform airstream at different heights above a hill. The stippled areas are regions of downward displacement. The centre of the hill is marked and one of the contours is shown by a dashed line (except in Fig. 5, only half the pattern is shown in each case because it is symmetrical).

In each case the pattern is shown for $lz = 1, 2, 3, 4$. The quantity $l = (g\beta)^{\frac{1}{2}}/U$, typically has a value in the region of 1 km^{-1} in the troposphere, so that the diagrams may, for example, be taken to indicate the displacement at heights of 1, 2, 3, 4 km. The height of the hill is in arbitrary units, the displacement being in the same units. Thus comparing Figs. 3 (a) and 4 (a) it is seen that although the circular hill is more than twice as high as the oval one the vertical displacement is not much greater. If the two hills were of the same height the steeper-faced, more nearly two-dimensional, oval hill would produce about twice the vertical displacement. The heights and displacements may be conveniently thought of as metres or feet.

In all diagrams the wind is from left to right unless the other independent solution is chosen, in which case the direction of flow is reversed.

Close to the dotted line the integral was not evaluated and the contours were simply drawn across from one side to the other. This dotted line is shown only in Fig. 3, but would occupy the same position for the same value of lz in Figs. 4 to 6.

3. Characteristics of the flow

The results are displayed in Figs. 3-6 which show the contours of vertical displacement. The formulae of Section 1 are derived by taking the same solution as in the previous calculations in I–III. If the other independent solution discussed in III, and formerly

favoured by Queney and others, is taken, then the wind direction is simply reversed. For a more practical case (i.e., not a uniform airstream), the two solutions must be added in proportions determined by the case. In many practical cases lee waves are also likely and these have not yet been computed for any non-two-dimensional hill. The diagrams serve, however, to illustrate the possibilities. Steepness and obliqueness obviously have considerable effect; but, remembering that the case of a steady uniform airstream is rather special, one should merely be informed by these diagrams and should be careful about extrapolation to more complicated cases.

Acknowledgments

Most of the calculation was done by Mrs. M. Wilkinson and the drawing by Miss S. A. Latta, and to them I am most grateful.

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1954 III *Ibid.*, **80**, p. 417.

551.510.42 : 621.385.833

The study of air pollution with the electron microscope

By J. CARTWRIGHT, G. NAGELSCHMIDT and J. W. SKIDMORE
Safety in Mines Research Establishment, Sheffield

(Manuscript received 27 October 1955)

Summary

The use of electron microscopy for observation and measurement of solid particles in air pollution is described, the thermal precipitator being used as sampling instrument. By using heat-stable membranes, it is possible to distinguish combustible from non-combustible matter, and other chemical tests can be used to distinguish different materials.

The shape of the particles in air pollution is mainly spherical; the size varies from 0·01 to 1 μ, although occasional larger particles are found. Many of the spheres are aggregated and resemble carbon black.

Examples are given of quantitative evaluation in terms of weight per unit volume for different size classes. The few results obtained so far show reasonable agreement with previous mass estimates based on smoke-filter sampling. The future developments in the use of the electron microscope for studying air pollution are briefly discussed.

1. Introduction

In research on coal miners' pneumoconiosis, airborne dust particles small enough to reach the interior parts of the lung are studied in this laboratory. The most powerful modern research tool that can be used in this field is the electron microscope. It has a far higher resolving power than the light microscope and is therefore particularly suitable for studying solid particles down to a few 100 Å (1 Å = 10^{-8} cm) in size. Above a size of 2 or 3 μ there is no need to use an electron microscope as the resolving power of the optical microscope is sufficient. Our studies were, therefore, made with an optical microscope for the larger particles and with an electron microscope for the particles of less than 2 μ diameter.

In the course of this work evidence of air pollution was found in almost every sample taken underground or in the laboratory, a few tests were made to follow this observation, and the present paper describes our results to date.

Although our work is mainly concerned with the investigation of coal mine dusts, the results are equally applicable to the study of atmospheric pollution above ground. The mechanism of the harmful effects of air pollution on health is incompletely understood and the electron microscope is likely to be useful for further research in this field.

2. The sampling of aerosols of solid particles, and their appearance

In order to characterize solid particles in dust clouds it would be best to make measurements on the cloud itself, for instance by light-scattering techniques. With electron microscopy this is not possible; the particles must be collected from the air before they can be observed and counted. This collection should be done in such a way that the state of aggregation of the particles is altered as little as possible. Thermal precipitation is in our opinion the best mechanism and is likely to be superior to electrostatic precipitation or collection on membrane filters.

A technique of sampling aerosols by means of the thermal precipitator for electron microscopy has been developed in this laboratory (Cartwright and Skidmore 1953). Formvar-coated cover slips are used instead of the ordinary glass slips; after sampling is finished the dust trace is metal-shadowed and can be first examined by light microscopy. The formvar film is then floated off and mounted on a specimen grid with the dust trace parallel to one set of grid lines. The specimen is now ready for examination in the electron microscope. Traverses of the dust trace for quantitative work can be made. Electron micrographs at known magnification, usually about × 3,000, are taken and evaluated by projection counting at an overall magnification of × 15,000.

When the electron microscope is used in this way to study airborne dust, it is found that there is a marked change in the character of the particles at about 1 μ size. Particles greater than 1 μ usually have angular outlines and the general appearance of broken mineral matter (Fig. 1, Plate III).

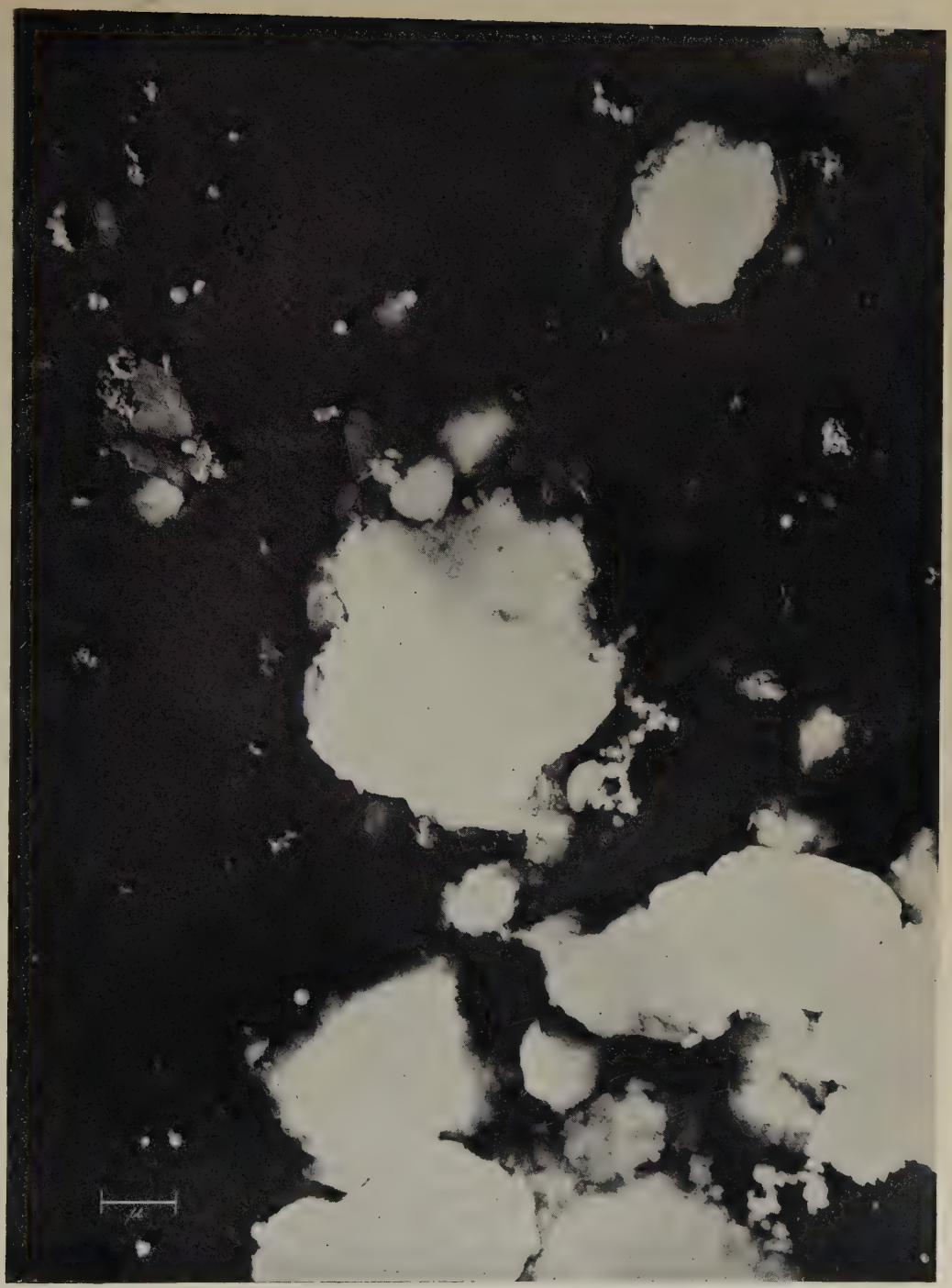


Figure 1. Dust in coal-mine air. (Crown copyright reserved).

Figure 2. Dust in intake air of coal mine. (Crown copyright reserved).

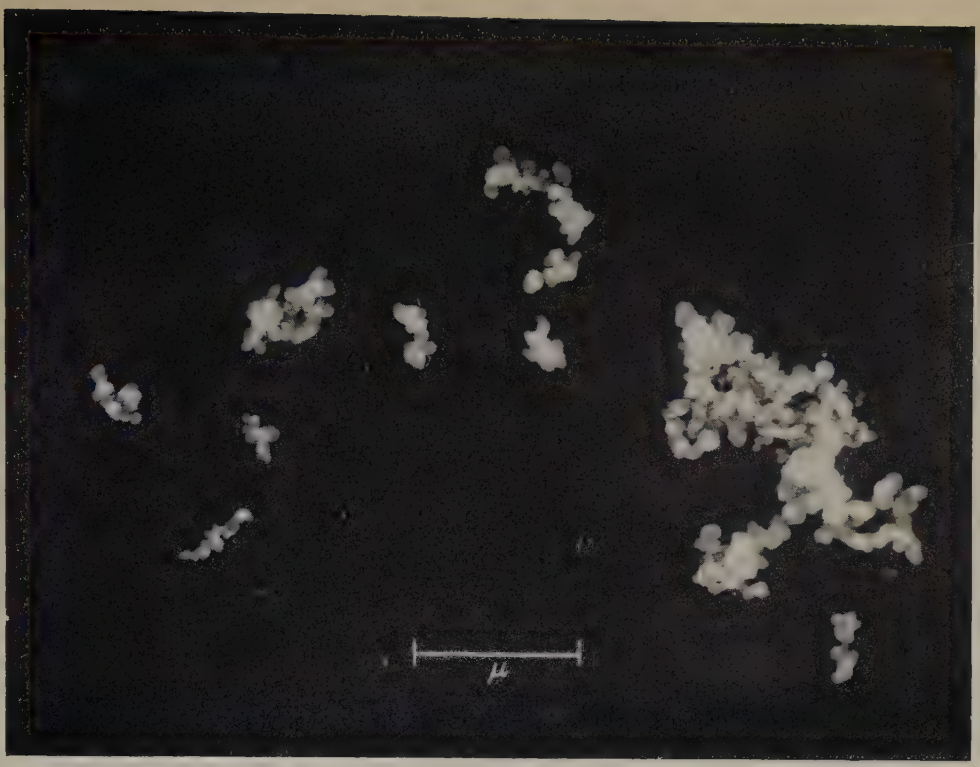


Figure 3. Carbon black (Statex B). (Crown copyright reserved).

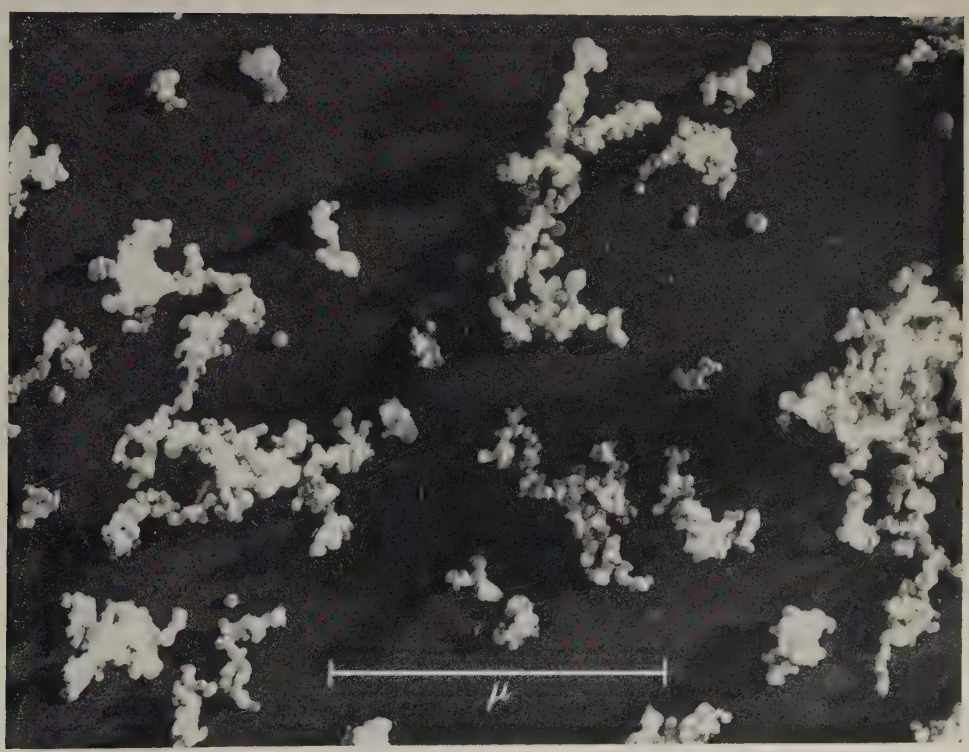


Figure 4. Diesel-smoke particles. (Crown copyright reserved).

Figure 5 (a). Particles in Sheffield air. Original sample. (Crown copyright reserved).

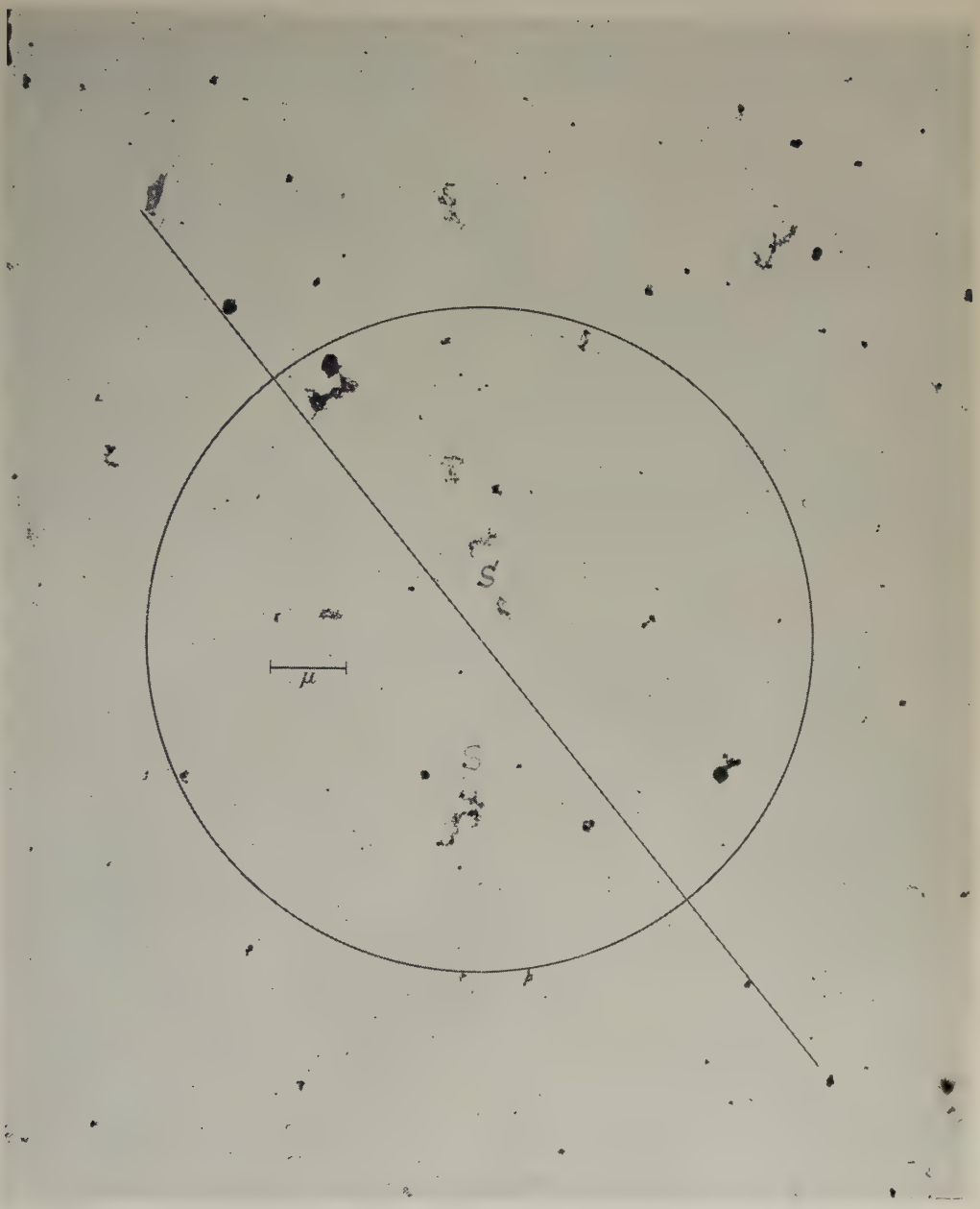


Figure 5 (b). Particles in Sheffield air. After heating in air to 400 °C. (Crown copyright reserved).

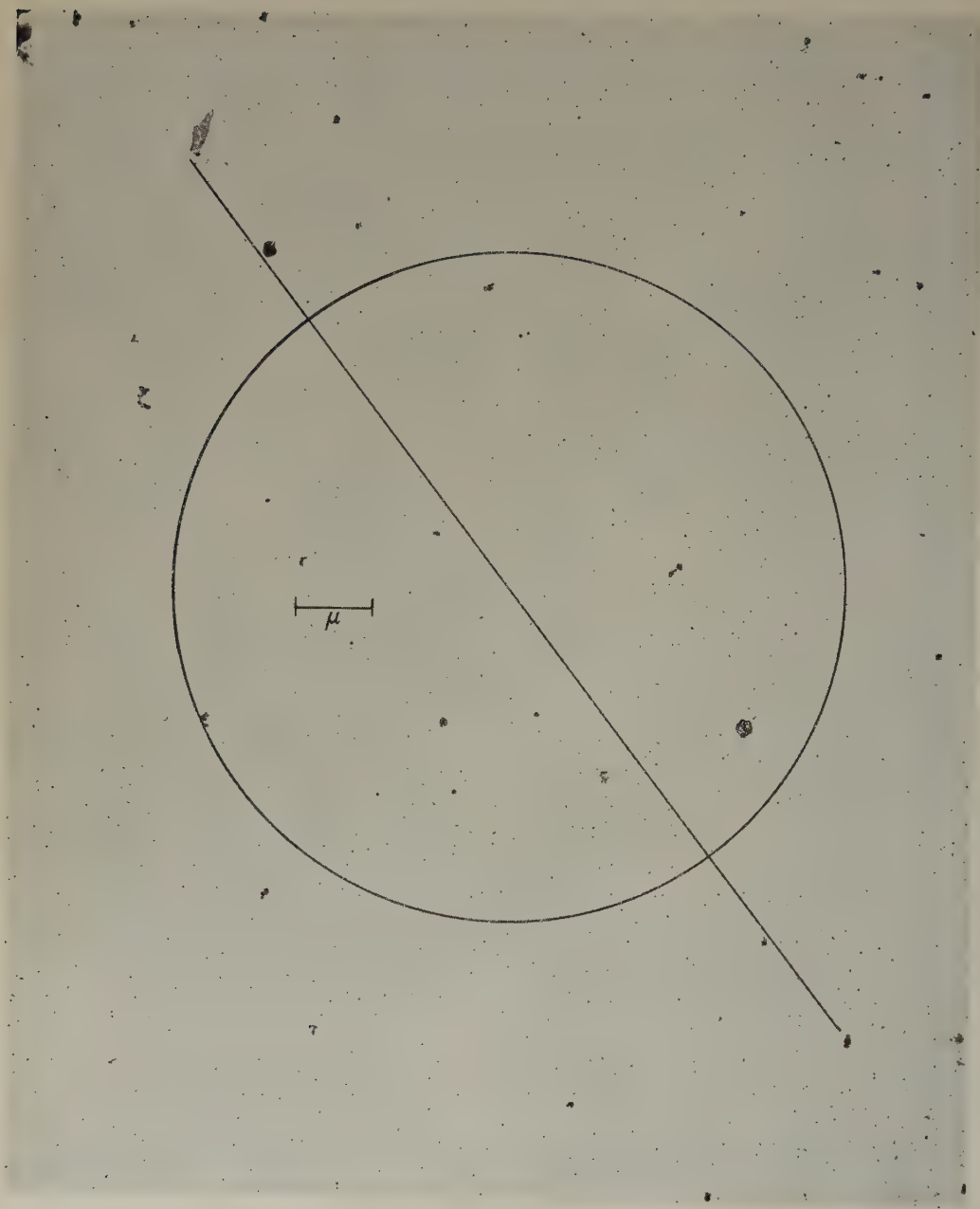
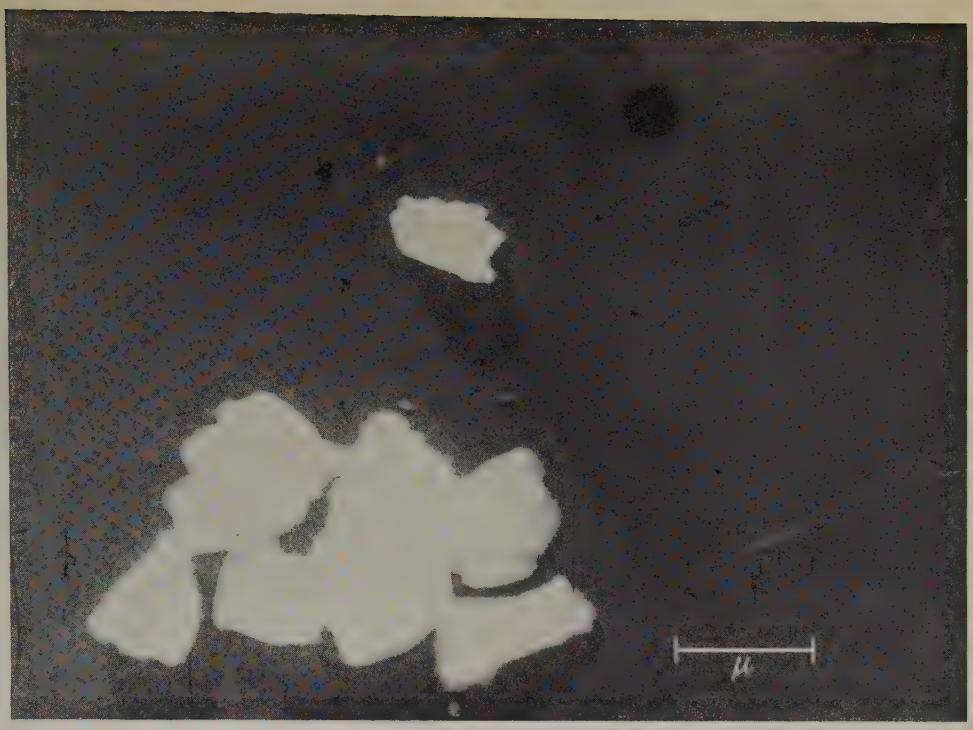
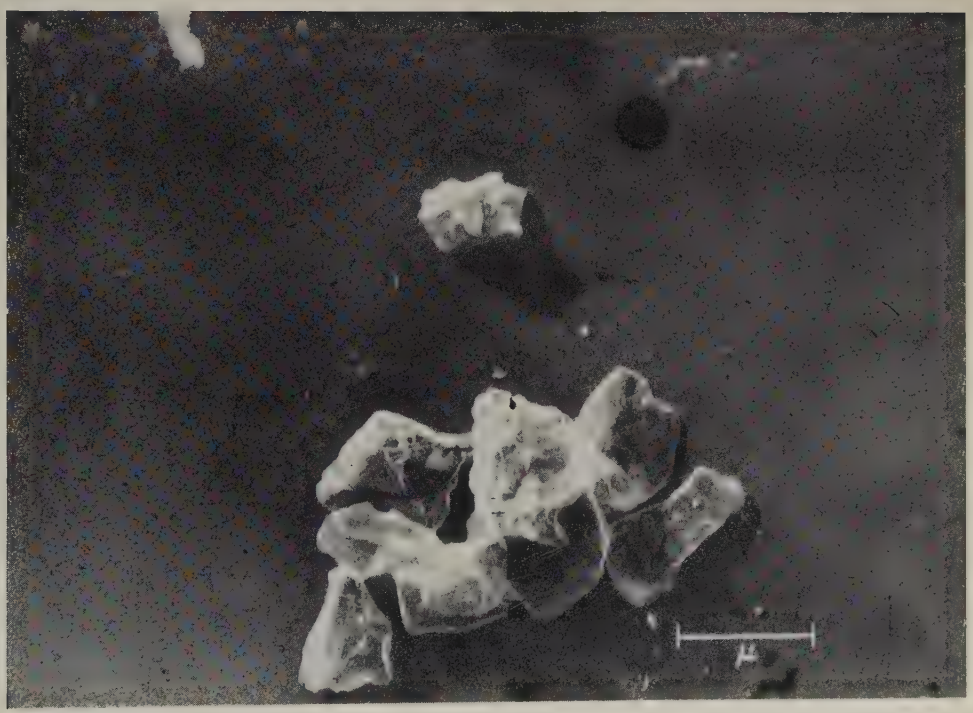


Figure 5 (c). Particles in Sheffield air. After heating in air to 550°C. (Crown copyright reserved).



(a)



(b)

Figure 6. (a) Quartz particles untreated. (b) The same particles treated with HF. (Crown copyright reserved).

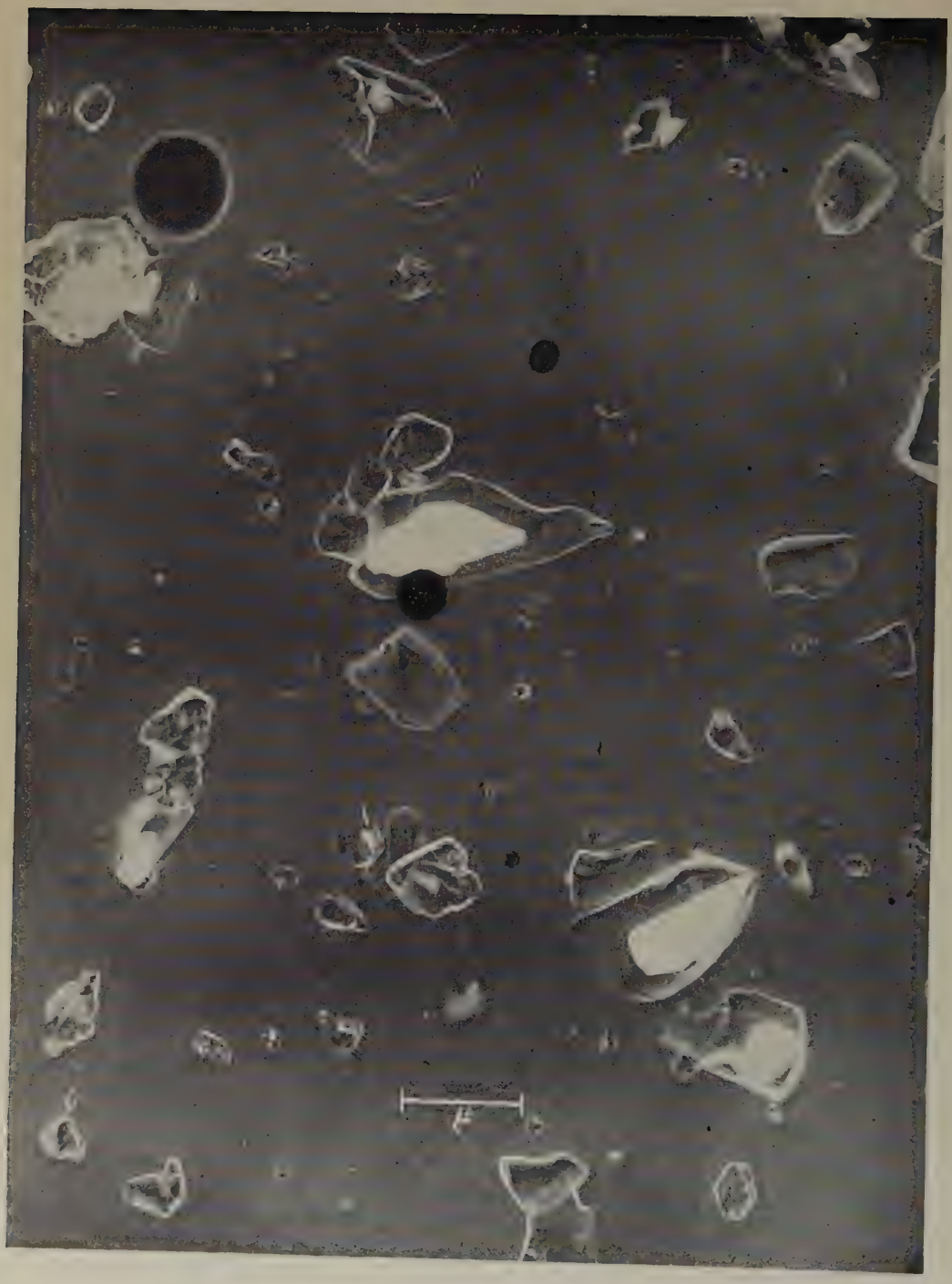


Figure 7. Quartz particles partially removed by HF vapour. (Crown copyright reserved).

Figure 8. Dust sample in Sheffield atmosphere, showing shallow craters. (Crown copyright reserved).

Below 1 or 2 μ, however, only few of the particles are angular in outline. The most commonly occurring shapes are near-spherical single particles and loose aggregates of spheres (Fig. 2, Plate IV). This general difference in appearance between the particles in the optical size range and those mainly in the electron microscope range is very striking. The spherical particles below 2 μ resemble carbon black (Fig. 3, Plate V), or diesel smoke (Fig. 4, Plate V), in their shape and density and often in their state of aggregation, but the constituent spheres in air pollution cover usually a wider range of sizes than carbon black or diesel smoke, and there may be many single spheres present.

3. The nature of the particles

Whether the spheres in air pollution consist of carbon or fused mineral matter can neither be decided from their appearance nor from electron-diffraction tests, as they usually all give only amorphous bands indicative of random structure and no diffraction spots or sharp rings indicative of crystalline material. But by sampling on heat-stable membranes of alumina instead of the normal formvar membranes it is possible to determine the combustion temperature of any carbonaceous matter. Figs. 5 (a), (b) and (c) (Plates VI, VII and VIII), show a sample of air pollution taken in the summer in the centre of Sheffield; (a) is the original sample, (b) the same field of particles after heating for 30 min in air at 400°C, and (c) the same field after similar heating at 550°C. These temperatures were chosen because the rate of oxidation of carbon black particles of 500 A diameter, when heated in a thermal balance, was very low at temperatures below 500°C but rose steeply above this value.

The aggregate of spheres marked *H*, and most of the single spheres of 0·05 to 0·5 μ diameter shown in Fig. 5 (a), have disappeared after heating to 400°C. The remaining chains of particles of 0·05 μ or less, marked *S* on Fig. 5 (b) were unaffected by heating to 400°C but were entirely removed by further heating to 550°C (Fig. 5 (c)). It is likely that most of the particles in this sample were more tar-like (*H*), a smaller proportion were more like carbon black (*S*) and the remainder shown in Fig. 5 (c) were mineral matter or ash.

Further work is required to make these distinctions more precise, but the observations illustrate the usefulness of the heat-stable membranes in the study of air pollution by electron microscopy.

Other tests for chemical composition are also possible. For instance, the amount of quartz in an electron microscope sample taken on a formvar membrane can be estimated by examining it before and after treatment with hydrofluoric acid vapour (HF).

If the specimen has previously been subjected to a sufficiently intense electron bombardment during examination in the electron microscope before treatment, the site of deposition, and the size of the quartz particles, is indicated by a clearly visible outline on the membrane. This is the remainder of a thin carbon shell formed during electron bombardment from the trace of hydrocarbon vapour contaminant in the vacuum system of the microscope. Fig. 6 (a) (Plate IX) shows a group of quartz particles before, and Fig. 6 (b) (Plate IX), their outlines after HF treatment; Fig. 7 (Plate X) shows partially removed particles.

Sodium chloride can be evaporated in the electron microscope by sufficiently intense illumination (Walkenhorst 1952). It is likely that a variety of similar techniques can be developed for identifying other chemical compounds.

Although the electron microscope is unsuitable for a direct study of liquids and gases, evidence of their existence may be found on the micrographs. Whenever samples were taken in misty weather the micrographs showed circular rings which could represent the perimeters of deposited and subsequently evaporated droplets (Dautrebande and Walkenhorst 1953).

The sample shown in Fig. 8 (Plate XI) was taken on the roof of the Sheffield laboratory when the atmosphere was hazy and visibility was about 500 yd. The areas within the rings are seen, after metal shadowing, to form shallow craters with often a single small particle in each crater. It is probable that the craters mark the site of deposition of liquid droplets and that the particles in the craters are either condensation nuclei or material originally in solution in the droplets.

4. Quantitative analysis

In the evaluation of thermal precipitator samples with the light microscope, the projected areas of the particles are compared with a series of circles of different diameter, and numbers of particles of different size classes per unit volume of air sampled are obtained. The same procedure, using projection counting, can be used when dust samples are examined with the electron microscope (Cartwright 1954). We have adopted the convention of counting aggregates

as single particles, mentally compressing them to the size they would have if the free space inside the aggregates were eliminated. It is sometimes possible also to distinguish denser and less dense particles of a given size.

From the particle-number distribution the surface and mass distributions can be calculated, and as nearly all single particles in air pollution appear to be spherical no special shape factor needs to be introduced. The shape of the aggregates is sometimes far from being spherical, but the above convention was extended to treat them as spheres and this is not likely to introduce a serious error. Examples of the kind of information that can be obtained in this way are given in Table 1. The six locations chosen represent contrasted environments. They were :

1. Country atmosphere. Sample taken at Harpur Hill, near Buxton, in December 1954.
2. Industrial city atmosphere. Samples taken on the Safety in Mines Research laboratory roof near the centre of Sheffield in February 1955.
3. South Wales. Sample taken of intake ventilation air to a bituminous coal mine.
4. Sample taken underground on the return of a coal face in the same coal mine.
5. South Wales. Sample taken of ventilation air to an anthracite coal mine.
6. Sample taken on the return of a coal face at the same coal mine.

Samples 3, 4, 5 and 6 were taken in March and May 1955.

TABLE 1. PARTICLES/CM^3 ESTIMATED FROM ELECTRON-MICROSCOPE COUNTS OF THERMAL-PRECIPITATOR SAMPLES

			South Wales			
			Bituminous coal		Anthracite	
Size-range (μ)	Buxton	Sheffield	Surface	Underground	Surface	Underground
	1	2	3	4	5	6
< 0·06	400	6140			904	
0·06 to 0·08	510	9100	2762	410	872	150
0·08 to 0·11	1370	11900	2050	386	1390	127
0·11 to 0·16	820	6320	2020	447	602	206
0·16 to 0·23	550	3380	1110	236	545	95
0·23 to 0·32	320	2230	965	162	351	110
0·32 to 0·45	150	1560	875	162	182	95
0·45 to 0·64	80	700	455	62	94	63
0·64 to 0·90	20	286	245	50	53	63
0·9 to 1·3	20	130	69	62	3	79
1·3 to 1·8	10	26	34	25		80
1·8 to 2·6			5	37		81
2·6 to 3·6				25		51
3·6 to 5·1				12		30
No. of particles actually counted	425	1549	2180	327	2977	315

It is clear from the results shown in Table 1 that in most cases the numbers of particles below the limit of optical resolution (*c.* 0·2 μ) greatly exceed the numbers of those within the optical range. In the Sheffield atmosphere 33,000 particles smaller than 0·16 μ were estimated to be in each cm^3 of air. The numbers alone, however, convey little information about the physical characteristics of the airborne material and it is desirable to calculate the mass or surface they represent. This is done in Table 2, where the sampling sites are rearranged in order of increasing total mass. The particle density was taken to be 1·6. This may underestimate the weight of ash or mineral particles whose density would exceed 1·6.

In order to see whether these results were at least of the right order of magnitude, they were compared with the estimated weight of smoke from smoke filters in equivalent localities, given by Meetham (1952). The results are given at the bottom of Table 2 and it appears that our electron microscope estimate of 88 μg/m^3 for the Buxton sample lies between the values of 60 and 100 μg/m^3 as estimated by smoke filters for Leicestershire. Similarly, the electron microscope value of 570 μg/m^3 for Sheffield lies between the smoke-filter values of 420 μg/m^3 for Leicester and 660 μg/m^3 for Glasgow.

Although this shows reasonable agreement, it is not permissible at this stage to draw any definite conclusions from the results so far obtained; it is known that very large daily and seasonal variations occur in the amount of air pollution, and the electron microscope values were obtained from snap samples.

TABLE 2. CALCULATED WEIGHT ($\mu g/m^3$) OF ALL PARTICLES SMALLER THAN 5 μ.

Size range (μ)	5 S. Wales (anthracite) surface	2 Buxton	3 S. Wales (bituminous) surface	1 Sheffield	4 S. Wales (bituminous) underground	6 S. Wales (anthracite) underground
< 0·06				0·53		
0·06 to 0·08	0·23	0·13	0·6	2·4	0·13	0·04
0·08 to 0·11	1·1	1·1	1·5	8·7	0·27	0·13
0·11 to 0·16	1·2	1·7	4·2	13	0·93	0·40
0·16 to 0·23	3·2	3·2	6·5	20	1·3	0·53
0·23 to 0·32	5·8	5·3	16·0	37	2·7	1·9
0·32 to 0·45	8·6	7·1	41	74	7·7	4·5
0·45 to 0·64	12·4	10·6	61	93	8·2	8·3
0·64 to 0·9	19·6	7·4	91	106	19	23·3
0·9 to 1·3	3·2	21·2	73	140	66	83·6
1·3 to 1·8		30·0	102	78	74	240·0
1·8 to 2·6			42		313	686·0
2·6 to 3·6					600	1220·0
3·6 to 5·1					814	2040·0
Total weight $\mu g/m^3$	56	88	440	570	1910	4310

	Leicestershire			
Comparable estimates by smoke filter*	In east winds	In all winds	Leicester	420
	60	100	Glasgow	660
			London	840

* Data quoted by Meetham (1952), p. 187.

The data given in Table 2 show a remarkable similarity between the weight distributions of airborne dust in the Buxton and the South Wales anthracite region for particles smaller than 0·64 μ. There are differences for particles of larger size but these could be due to random fluctuations, as the actual numbers of particles counted were small. This similarity, and the low total weight in either situation, could be explained by their remoteness from a region where large quantities of bituminous coal are burnt.

There is again a similarity between the total weight of suspended matter in Sheffield and in the surface air at a bituminous coal mine in South Wales. Although these sites are well separated geographically, a common feature could be the large consumption of bituminous coal in each locality.

In the case of the two collieries investigated, particles smaller than 0·64 μ were more abundant in the open air than they were in the return air from the coal face. The effect is shown in the tables and in Figs. 9 and 10. In the bituminous coal mine the weight of dust in the ventilation intake was 23 per cent of the total dust of respirable size found underground, but very little of this intake dust, of particle size less than 1 μ, was found in the return air from the coal face. The amount of dust in the intake air to the anthracite mine was very small but the apparent loss of fine particles below ground was still demonstrable.

It is not known whether this effect is due to an actual loss of fine particles from the air, or whether these particles remain, but are obscured by the larger particles to which they are attached by impingement in the air or during collection in the thermal precipitator. The effect is considerable and needs further investigation.

DISCUSSION

The new techniques described have not been used sufficiently to warrant a long discussion. Much further work needs to be done to develop specific methods for different constituents of air pollution. The quantitative results obtained appear reasonable, but as the technique is laborious, we believe that one of the main tasks of electron microscopy in this field will be to calibrate simple methods which can be used in routine analysis. It is not known, for instance, how much smoke penetrates the filter paper used in the standard D.S.I.R. smoke-filter test, and how penetration varies with the loading of the filter. The electron-microscope techniques will probably remain essentially research techniques, even if the development of automatic evaluation of the records (Walton 1954) replaces the projection counting used at present.

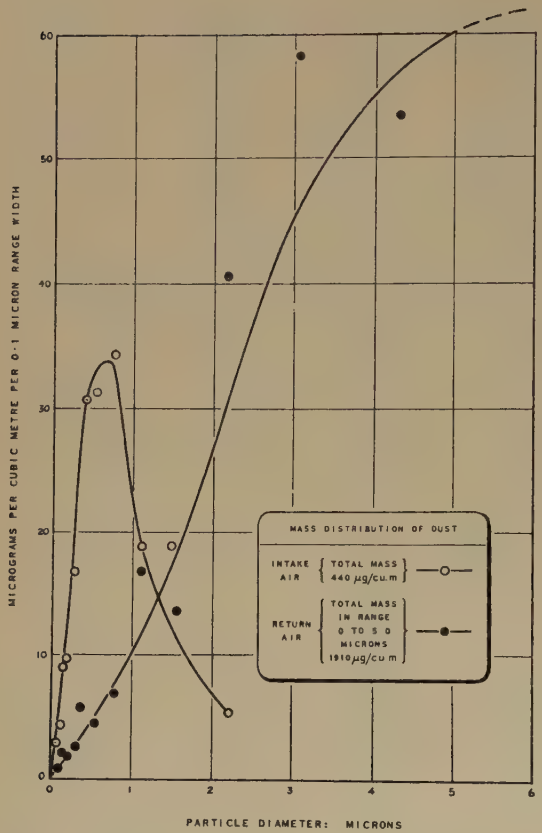

MICROGRAMS PER CUBIC METRE PER 0·1 MICRON RANGE WIDTH
PARTICLE DIAMETER: MICRONS
MASS DISTRIBUTION OF DUST
INTAKE AIR { TOTAL MASS 440 µg/cu.m
RETURN AIR { TOTAL MASS IN RANGE 0 TO 5·0 MICRONS 1910 µg/cu.m

Figure 9. Mass distribution of dust in intake and in return airways at a bituminous mine. (Crown copyright reserved).

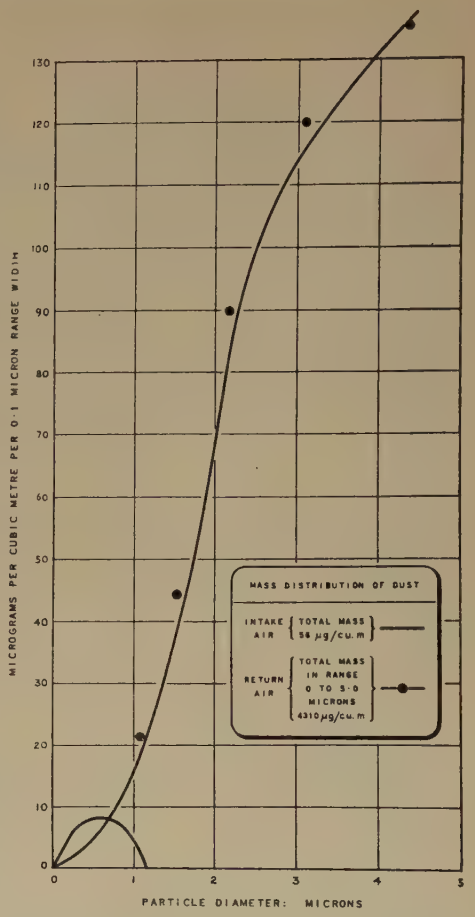

MICROGRAMS PER CUBIC METRE PER 0·1 MICRON RANGE WIDTH
PARTICLE DIAMETER: MICRONS
MASS DISTRIBUTION OF DUST
INTAKE AIR { TOTAL MASS 56 µg/cu.m
RETURN AIR { TOTAL MASS IN RANGE 0 TO 5·0 MICRONS 4310 µg/cu.m

Figure 10. Mass distribution of dust in intake and in return airways at an anthracite mine. (Crown copyright reserved).

Acknowledgment

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551.501.7 : 551.557.36

A note on the parametric representation of non-circular wind distributions

By J. R. SCOTT
Malayan Meteorological Service, Singapore

(Manuscript received 2 August 1955, in revised form 8 November 1955)

SUMMARY

The theory and an example is given of a method of representing a non-circular wind distribution in which correlation exists between the departures from the means of the computed components. For it to be accurate the distribution of the components must be normal.

1. INTRODUCTION

The adequate representation of a wind distribution by the vector mean and the standard vector deviation requires the existence of the three conditions (see, e.g., Brooks and Carruthers 1953) (a) the distribution of the components in two directions mutually at right angles must be normal, (b) the scatter of each set of such components must be the same and (c) the departures from the means must be independent. Provided that condition (a) is satisfied it is possible to represent a non-circular distribution in which a significant correlation appears between the departures from the means of the components. This, as will be seen below, requires three parameters in place of the standard vector deviation.

2. THEORY

Let x' and y' be a pair of specific departures from the means in conventional axes, east and north respectively, and x and y the corresponding departures referred to axes rotated counter-clockwise through θ, the origin being at the extremity of the vector mean. Then

$$x = x' \cos\theta + y' \sin\theta, \qquad y = y' \cos\theta - x' \sin\theta,$$

and it is readily deduced that

$$\sigma_x^2 = \sigma_{x'}^2 \cos^2\theta + \sigma_{y'}^2 \sin^2\theta + 2\rho' \sigma_{x'} \sigma_{y'} \sin\theta \cos\theta,$$

$$\sigma_y^2 = \sigma_{y'}^2 \cos^2\theta + \sigma_{x'}^2 \sin^2\theta - 2\rho' \sigma_{x'} \sigma_{y'} \sin\theta \cos\theta,$$

$$\rho \sigma_x \sigma_y = \rho' \sigma_{x'} \sigma_{y'} (\cos^2\theta - \sin^2\theta) + (\sigma_{y'}^2 - \sigma_{x'}^2) \sin\theta \cos\theta \quad . \quad . \quad (1)$$

$\sigma_{x'}$, $\sigma_{y'}$, σ_x and σ_y are the standard deviations of the components along the x', y', x and y axes respectively. ρ' and ρ are the correlation coefficients between the departures from the means of the components in the x', y' and x, y systems respectively. $\sigma_{x'}$, $\sigma_{y'}$ and ρ' being known we may select θ so that $\rho = 0$ and find σ_x and σ_y from Eqs. (1).

With $\rho = 0$, σ_x and σ_y are parameters derived from two sets of independent components, i.e., the probability (p) of any particular wind vector ending in the elementary area $\delta x\, \delta y$ located at (x, y) is given by

$$p = \frac{1}{2\pi \sigma_x \sigma_y} \exp\left[-\tfrac{1}{2}\left(\frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2}\right)\right] \delta x\, \delta y \quad . \quad . \quad . \quad (2)$$

Whence after integration it is found that the probability (P) of a vector ending in the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \text{where} \qquad \frac{a}{b} = \frac{\sigma_x}{\sigma_y}$$

is given by

$$P = 1 - \exp\left(-\frac{a^2}{2\sigma_x^2}\right) \quad . \quad . \quad . \quad . \quad (3)$$

Hence, provided only that the component distribution is sufficiently near normal, it is possible to represent the wind distribution by the parameters θ, σ_x and σ_y even though the distribution is not circular and correlation exists between the departures from the mean of the components originally extracted.

3. Example

The 03 GMT 50,000 ft wind at Singapore was observed on 86 occasions during the months of September, October and November 1953. The mean wind was 86°, 32·9 kt, $\sigma_x' = 20{\cdot}9$ kt, $\sigma_y' = 9{\cdot}0$ kt and $\rho' = +0{\cdot}30$. These figures yielded $\theta = +8{\cdot}8°$, $\sigma_x = 21{\cdot}0$ kt and $\sigma_y = 8{\cdot}5$ kt. The probability (Mauchly's test, described by Brooks and Carruthers 1953) of the distribution occurring in a sample from a circularly distributed population is 10^{-7}. Of the 86 observations available it was possible to extract 44 separated from one another by at least 48 hr. Each set of the originally extracted components of these was regarded (Brooks and Carruthers 1953) as independent, and their distributions compared with those of normal distributions having the means and standard deviations computed from the 86 observations. Five intervals of deviation from the means were used (limited by the four points $\pm\sigma'$ and $\pm\frac{1}{2}\sigma'$). The χ^2 test showed that the hypothesis that the sets of components were random samples from normally distributed populations having the parameters computed, was not untenable. The probabilities corresponding to the values of χ^2 found both substantially exceeded 0·10.

The wind vector ends were plotted together with the ellipses which could be expected to contain 25, 50, 63, 75 and 85 per cent of them. These ellipses contained 26, 47·5, 61·5, 73 and 89·5 per cent of vector ends respectively. To examine the distribution of direction, lines inclined at $\pm\tan^{-1}\sigma_y/\sigma_x$ to the x axis were drawn. This divided the diagram into eight sectors in each of which an equal number of vector ends were to be expected. For the purpose of comparison similar lines (at 45° to the axes) corresponding to a circular distribution were constructed. Table 1 compares the number of vector ends contained in the eight sectors required by the elliptical distribution with those required by the corresponding circular distribution. The first column contains frequencies of vector ends in the first sectors moved over by a counter-clockwise moving vector rotating about the centre of the ellipses and circles, starting from the positive x axis. The second contains those in the second sectors, etc.

TABLE 1. Distribution of direction of 03 GMT, 50,000 ft wind at Singapore

Expected	All 10·75								Total
Ellipse sectors	12	9	13	13	9	14	7	9	86
Circle sectors	14	7	7	19	16	7	5	11	86

4. Conclusion

The sample chosen for the example is a small one. However, similar results have been obtained from a number of other larger samples obtained from different heights and seasons (none of these was investigated graphically). It is to be concluded therefore that nearly elliptical distributions do exist, and that their comparatively simple parametric representation is possible and reasonably accurate.

Acknowledgments

I am indebted to the Director of the Malayan Meteorological Service for permission to publish this note. The data used for illustration were obtained from a research programme financed by the Colonial Welfare and Development Fund.

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551.511 : 551.558.1 : 551.524.4

Some measurements of heat flux at large negative Richardson number

By R. J. TAYLOR

Division of Meteorological Physics, Commonwealth Scientific and Industrial Research Organization, Melbourne, Australia

(Manuscript received 5 September 1955)

SUMMARY

Some measurements of heat flux, H, at large negative Richardson number confirm and add precision to the conclusion of Priestley (1955) that heat is transported effectively by a process of free convection over a wide range of conditions. In this range $H \propto (\partial\theta/\partial z)^{3/2}$ and the measurements lend weight to the suggestion that this regime exists when $-Ri_{1\cdot5\,m}$ exceeds about 0·02.

1. INTRODUCTION

In conditions of strong lapse and small wind shear, it has been shown by Priestley (1955) that $\overset{*}{H}$, given by

$$\overset{*}{H} = \frac{H}{\rho\, c_p \left(\frac{g}{\theta}\right)^{1/2} \left|\frac{\partial\theta}{\partial z}\right|^{3/2} z^2}$$

(where H is the vertical turbulent flux of total heat, θ is the potential temperature, and the other symbols have their usual meanings) can attain a constant value characteristic of a regime of free convection and independent of the Richardson number. The observations which he analysed indicated that this regime existed when the Richardson number measured at 1·5 m, $Ri_{1\cdot5}$, had a negative value greater than about 0·02 and that the most probable value of $\overset{*}{H}$ was then 0·81. Further observations bearing on these conclusions have now been made.

2. THE OBSERVATIONS

Twenty-two measurements of heat flux and associated wind and temperature gradients were made over observation periods of 5 min following the techniques already described by Swinbank (1955) and McIlroy (1955). Concentration of effort on days of very light wind and clear sky permitted the range of $|Ri|$ greater than 0·02 to be increased considerably (a forty-fold range as against a twelve-fold range in the analysis referred to) with a consequent significant improvement in the precision with which conclusions can be drawn.

The observations are presented in Table 1. In addition to the quantities of immediate interest, the table also includes measurements of mean wind speed at 2 m and net incoming radiation (where available) and the calculated values of root-mean-square fluctuation in temperature and vertical wind component, and of the coefficient of correlation between these two. The highest value of $|Ri|$ shown, 0·865, to some extent represents an instrumental limitation; a few occasions did occur with greater $|Ri|$ but the wind was then so light that large variations in azimuth occurred and the vane was unable to keep the flux-measuring apparatus satisfactorily headed into wind. Such occasions, however, were rare.

3. DISCUSSION

The present set of measurements has been taken together with those used by Priestley for which $|Ri| \geqslant 0{\cdot}02$ in order to calculate the regression of $\overset{*}{H}$ on $|Ri|$. A regression equation of the form.

$$\log \overset{*}{H} = a \log |Ri| + b$$

was assumed and the calculated values of the coefficients (using logarithms to base 10) are :

TABLE 1. OBSERVATIONS MADE AT LARGE NEGATIVE RICHARDSON NUMBER

Date	Time EST	Height (m)	$\bar{u}$ at 2 m (m sec^{-1})	Net incoming radiation (m watt cm^{-2})	$\left(\frac{\partial \bar{u}}{\partial z}\right)_{1\cdot5\,m}$ (sec^{-1})	$\left(\frac{\partial \bar{\theta}}{\partial z}\right)_{1\cdot5\,m}$ (°C m^{-1})	$Ri_{1\cdot5\,m}$	σ_w (cm sec^{-1})	$\sigma_T \times 10^2$ (°C)	r_{wT}	H (m watt cm^{-2})	$\overset{*}{H}$
21.12.54	1317	7	2·38	66·1	0·387	− 0·842	− 0·184	57	54	0·57	20·1	0·54
	1336	7	2·63	63·3	0·381	− 0·819	− 0·186	38	43	0·45	8·2	0·23
	1341	7	1·85	62·5	0·273	− 0·646	− 0·288	50	53	0·48	14·2	0·56
10.1.55	1246	1·5	1·47	61·3	0·227	− 0·676	− 0·439	40	94	0·34	15·2	0·55
	1254	1·5	2·10	61·0	0·347	− 0·975	− 0·272	30	104	0·52	18·9	0·40
	1306	1·5	2·45	59·7	0·364	− 0·715	− 0·182	31	87	0·49	15·6	0·52
8.3.55	1224	1·5	2·17	47·5	0·250	− 0·328	− 0·176	21	68	0·46	8·0	0·86
11.3.55	1027	1·5	1·94	39·9	0·249	− 0·374	− 0·201	22	68	0·59	10·6	0·94
	1137	1·5	2·00	47·0	0·282	− 0·438	− 0·181	22	77	0·56	11·5	0·81
18.3.55	1145	1·5	1·94	43·1	0·248	− 0·341	− 0·185	22	82	0·57	12·4	1·26
	1201	1·5	1·04	43·3	0·121	− 0·381	− 0·865	23	93	0·37	9·4	0·80
	1216	1·5	1·12	43·6	0·114	− 0·311	− 0·803	23	81	0·59	13·0	1·52
22.3.55	1019	1·5	1·70	33·0	0·138	− 0·459	− 0·806	17	55	0·53	5·9	0·39
	1100	1·5	1·45	37·5	0·174	− 0·369	− 0·403	21	78	0·50	9·8	0·89
	1121	1·5	1·66	39·6	0·231	− 0·326	− 0·203	23	70	0·56	10·9	1·21
9.5.55	1127	4	2·00	not recorded	0·197	− 0·256	− 0·226	26	42	0·46	6·3	0·93
10.5.55	1059	4	1·33	not recorded	0·188	− 0·260	− 0·251	19	32	0·57	4·3	0·62
	1112	4	1·78	not recorded	0·161	− 0·282	− 0·371	18	35	0·70	5·7	0·73
	1122	4	1·33	not recorded	0·134	− 0·326	− 0·616	19	28	0·73	4·8	0·50
	1131	4	1·03	not recorded	0·104	− 0·185	− 0·586	22	35	0·53	5·0	1·21
	1141	4	0·97	not recorded	0·104	− 0·202	− 0·642	21	35	0·64	5·8	1·23
	1208	4	1·73	not recorded	0·189	− 0·256	− 0·244	23	31	0·54	4·8	0·71

Symbols not explained in text : $\bar{u}$ = mean horizontal wind velocity (measured by cup anemometer), σ = root-mean-square deviation from a mean, r is a coefficient of correlation. Subscript w refers to vertical velocity component, subscript T to temperature.

(a) for all observations (76 in number)

$$a = 0{\cdot}07 \pm {\cdot}06,\ b = -\,0{\cdot}09 \pm {\cdot}09.$$

(b) for observations at low level only (50 in number)

$$a = -\,0{\cdot}05 \pm {\cdot}05,\ b = -\,0{\cdot}15 \pm {\cdot}07.$$

The 22 new measurements include 7 at a height of 4 m, which was not used as a height of observation in the previous series. These, together with 12 at 1·5 m, have been considered as being at a low level. The remaining three, at 7 m have been taken as high-level measurements.

These regressions support and improve the reliability of the conclusion that $\overset{*}{H}$ is independent of Ri over the range considered. The geometric mean values of $\overset{*}{H}$, the standard errors of which have also been considerably reduced, are

(a) for all observations $\overset{*}{H} = 0{\cdot}69 \pm {\cdot}05$.

(b) for observations at low level only, $\overset{*}{H} = 0{\cdot}79 \pm {\cdot}04$.

and the new observations provide no basis for commenting further on the suggestion that the difference between these two is probably due to five minutes being too short an observation period to give an adequate measurement of heat flux at the greater heights. If this is so, then the value of 0·79 for $\overset{*}{H}$ is to be preferred.

The present observations have no values of $|Ri|$ less than 0·176 and, for graphical presentation, they have therefore been taken together with the last group in Priestley's Table 1, which has Richardson numbers in the range of − 0·140 to − 0·240. The resulting 30 measurements have been divided into three groups and the last group in his table can now be replaced by the values given in Table 2. The corresponding changes have been made in his Fig. 1 and the revised version is shown as Fig. 1 of this paper.

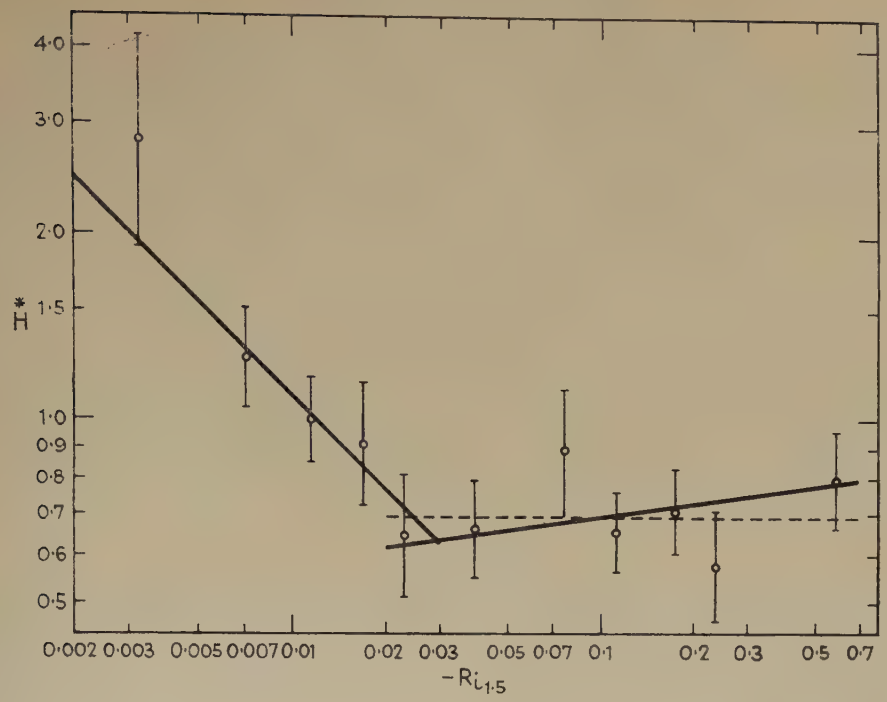

Figure 1. $\overset{*}{H}$ against $-Ri_{1\cdot5}$ for all observations. Mean values are shown by circles and the limits of standard (log) error by short lines. The full lines represent the calculated regressions and the broken line the geometric mean value of $\overset{*}{H}$ for $-Ri_{1\cdot5} \geqslant 0\cdot02$.

TABLE 2. $\overset{*}{H}$ IN TERMS OF Ri

Range of $\lvert Ri_{1\cdot5}\rvert$	$\geqslant 0\cdot14$ $< 0\cdot20$	$\geqslant 0\cdot20$ $< 0\cdot30$	$\geqslant 0\cdot30$
Number of measurements	11	10	9
Geometric mean $\lvert Ri\rvert$	0·174	0·234	0·589
$\overline{\log \overset{*}{H}}$	− 0·15	− 0·24	− 0·10
Standard error of $\overline{\log \overset{*}{H}}$	0·07	0·09	0·08

4. Conclusion

The observations which have been presented pertain to occasions of very large negative Richardson number which occur only rarely. Nevertheless, they strengthen markedly the conclusion that $\overset{*}{H}$ is independent of $\lvert Ri\rvert$ on all the much more frequent occasions when the latter is greater than 0·02. Confirmation has also been obtained, with increased precision, of the most probable value of the constant $\overset{*}{H}$. This value is close to 0·79.

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551.593.63

A tangent arc of the halo of $24\frac{1}{2}^{\circ}$

By S. W. VISSER and C. T. J. ALKEMADE
Royal Netherlands Meteorological Institute, De Bilt
Physics Department State University, Utrecht

(Manuscript received 4 July 1955)

SUMMARY

A remarkable halo complex observed in Germany in 1944 is described and theoretically discussed. There appear to be two possible crystal forms. Only one, however, with an inclination of the pyramidal faces of 25°14′ fulfills the theoretical conditions. As far as we know there is only one other observation of a tangent arc of an uncommon halo (in the Netherlands in 1933). It requires an inclination of the pyramidal face of 54°44′, just the angle measured by Clarke (1822). In both phenomena we are concerned with similar refraction in similar plate-shaped ice crystals.

1. INTRODUCTION

One of us (C.T.J.A.), then a student of the State University, Utrecht, Netherlands, being in compulsory civil service in Germany, had the good luck to see a remarkable halo complex. On the afternoon of 8 Nov. 1944, at 1615 LMT in the neighbourhood of Brunswick, he observed a double red-and-blue coloured ' tangent arc ' persisting for about 15 min, in a clearance after the passing of a depression. Cirrostratus and lower clouds were present. The direction of the observed arcs was between SW and W. The sun was covered by clouds. Besides the tangent arc a circular halo was faintly visible.

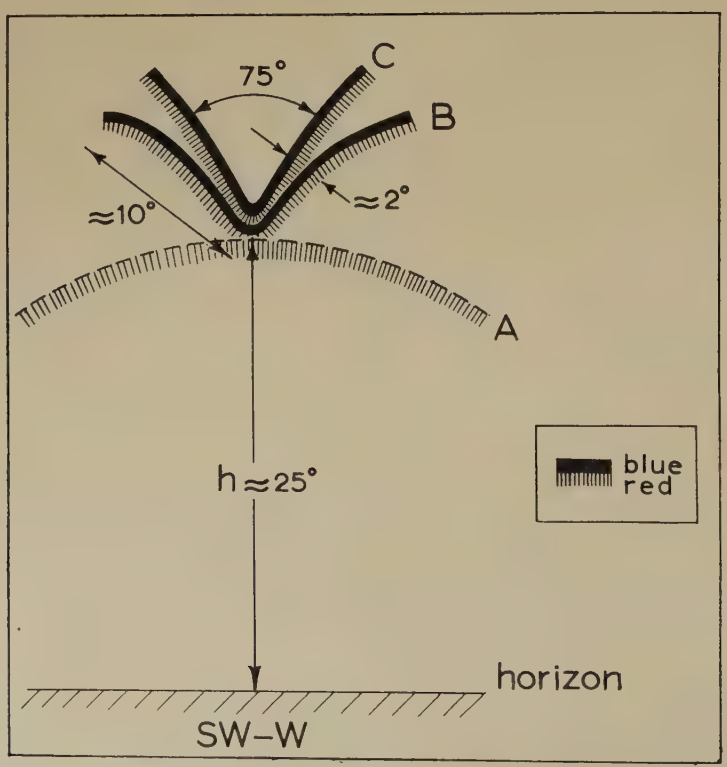

Figure 1. Drawing made during the observation, showing ' double tangent arc.' (Dr. Alkemade estimated the angles and the distances given, using outstretched fingers and knuckles – the only means at his disposal).

2. Theory

When discussing the phenomenon we agreed that the inconspicuous circle A was a part of the common 22° halo and the arc B, its upper tangent arc. It seemed reasonable to assume that the uncommon arc C was the tangent arc of the rare halo of radius about 24½°, so the shape of this arc was calculated.

The 24½° halo may develop when crystals with a refracting angle of 64°46′ are present. According to Hauy's law this angle is possibly dependent upon the base angle of 54°44′, measured by E. C. Clarke (1822) at Cambridge, 3 Jan. 1821.

The development of an upper tangent arc requires two conditions :

1. Rotating crystals with horizontal refracting edges. There are two possibilities concerning the inclination of the pyramidal faces with respect to the principal axis of the hexagonal crystal; 1a, inclination 64°46′; the refracting faces are a pyramidal face and the opposite prismatic face; 1b, inclination 25°14′, refraction at a pyramidal face and the opposite base (discussed in Section 4).

2. The rotating axis must be horizontal also. This means that in the two cases 1a and 1b this axis must be parallel to one of the major diagonals of the regular hexagonal section. Consequently, the crystals must principally be plate-shaped, as is also the case in the formation of the circumzenithal arc.

During the same afternoon (from 1200-1430 LMT) a remarkable display of circumzenithal arcs was observed in the Netherlands (R. Neth. Met. Inst. 1949). Many voluntary observers of the Meteorological Institute at De Bilt were impressed by the remarkable brilliancy of this arc and reported the occurrence in aircraft condensation trails. We suggest that under the extremely exceptional conditions of the cirrus clouds during their movement to the east the pyramidal parts developed and brought about the abnormal tangent arc above Brunswick.

3. Theoretical deduction of the shape of the tangent arc

In Fig. 2, Z = zenith, S = sun, O = observer, plane OAK = horizontal plane, H = orientation of refracting edge, BOZ = cross-section perpendicular to the edge OH (called ' normal plane '), K = pole of vertical plane through sun, AOZ, F = point of tangent arc. The length of the following arcs is denoted by a special symbol : AS = h = altitude of sun, CS = m, AG = a, BC = g, CD = k and p = difference in azimuth between planes AOZ and BOZ. The following constants will be used : n = index of refraction of crystals, e_0 = angle of refraction of crystals, r_0 = radius of the halo corresponding to the tangent arc to be discussed. Further, the position of F on the tangent arc will be determined by : EF = x and ES = $r_0 + y$. These coordinates are preferred for reason of convenience in plotting the calculated results.

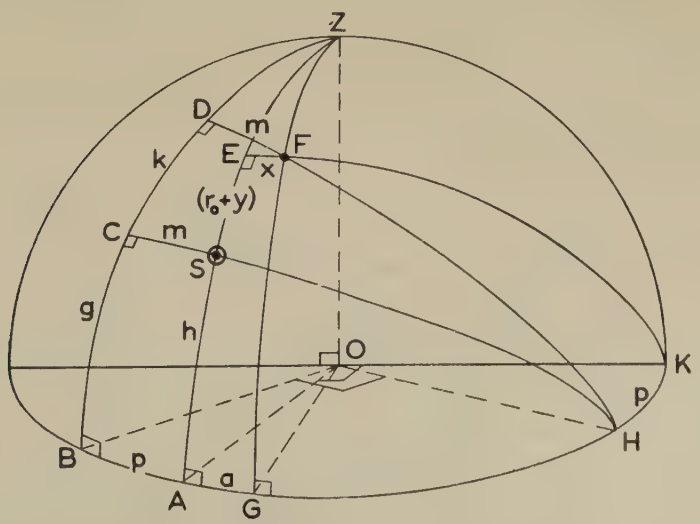

Figure 2. Hemisphere used in calculating the tangent arc.

Bravais (1847) proved that the minimum angle k between the projections on the normal plane, of the incident and refracted ray, respectively, can be calculated from the ' minimum deviation law ' if the refractive index is replaced by :

$$n' = n\,(1 - \sin^2 2m/n^2)^{\frac{1}{2}}/\cos m \quad . \quad . \quad . \quad . \quad (1)$$

It should be noted that in this expression m occurs as variable. Moreover, Bravais proved that the incident and refracted rays intersect the normal plane BOZ at equal angles. Thus DF = CS = m.

The calculation of the shape of the arc is made by taking for p several arbitrary values, and calculating at fixed values of h the corresponding values of g, m, k, a, y and x successively. Then y may be plotted as a function of x.

An explicit formula for k as a function of m, which holds with good approximation, can be derived by means of series expansions. This formula reads :

$$(k - r_0)\{1 - 0{\cdot}25\,(k - r_0)\tan\tfrac{1}{2}(r_0 + e_0)\} = 4\,(1 - 1/n^2)\tan\tfrac{1}{2}(r_0 + e_0)\sin^2\tfrac{1}{2}m\,(1 + \sin^2\tfrac{1}{2}m/n^2)/(1 - 2\sin^2\tfrac{1}{2}m) \quad (2)$$

For small values of m this expression may be simplified to :

$$(k - r_0) = 4\,(1 - 1/n^2)\tan\tfrac{1}{2}(r_0 + e_0)\,.\,\sin^2\tfrac{1}{2}m \quad . \quad . \quad (2a)$$

For higher values of m, $(k - r_0)$ may be calculated from Eq. 2 by successive approximation, the second term in the left-hand side of this equation being at first neglected. Since this term is small, its omission here can be easily corrected with sufficient accuracy in the second approximation.

The use of the above approximate formula may be advantageous, since it yields directly the *increment* $(k - r_0)$ as a function of m, since it admits a simple discussion of errors, and since the results obtained can be easily transformed for crystals with different angle of refraction or for different wavelengths. It should be noted that the variation of n with wavelength in the visible region may be disregarded in the small term $\sin^2\frac{1}{2}m/n^2$ in Eq. (2).

The results of calculation for $h = 0°$ and $5°$ are listed in Table 1 and sketched in Fig. 3 for the normal and abnormal tangent arcs respectively. The assumed value of n is 1·31 for yellow light.

TABLE 1. Results of calculations

Normal tangent arc, $e_0 = 60°$, $r_0 = 22{\cdot}0°$, $h = 5°$					
p	=	10°	20°	30°	40°
y	=	0·25°	1·0°	2·85°	4·8°
x	=	1·07°	2·4°	4·4°	7·7°
Abnormal tangent arc, $e_0 = 64°46'$, $r_0 = 24{\cdot}23°$, $h = 5°$					
p	=	10°	20°	30°	40°
y	=	0·4°	1·2°	3·0°	5·5°
x	=	1·25°	2·85°	5·07°	8·9°
Abnormal tangent arc, $e_0 = 64°46'$, $r_0 = 24{\cdot}23°$, $h = 0°$					
p	=	10°	20°	30°	40°
y	=	0·25°	1·25°	3·25°	6·0°
x	=	0·9°	2·1°	3·9°	6·9°

It was found later that the second-order terms occurring in the calculation influenced the value of $(k - r_0)$ by, at most, 7 per cent. The influence of the terms that are neglected in the series expansion is noticeably less, so it may be concluded that the accuracy of the results is of the order of a few per cent, which was sufficient in our case.

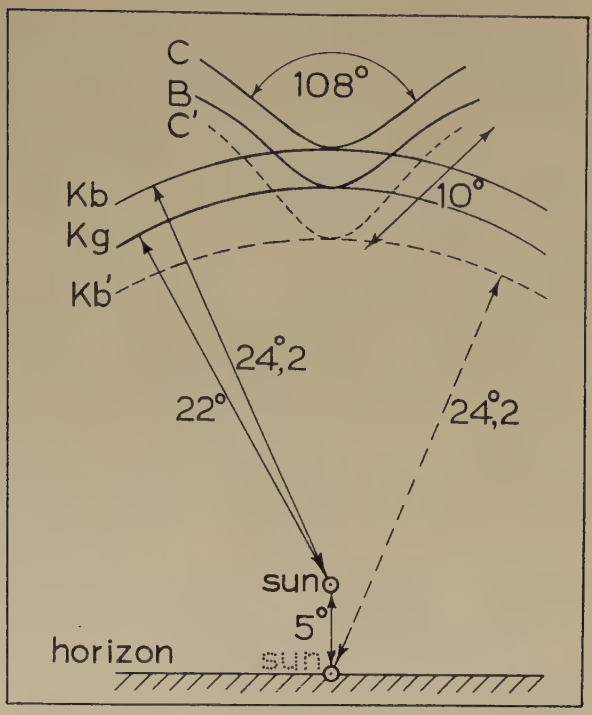

Figure 3. The calculated ' double tangent arc.'

Comparison between the calculated arcs (for altitude of sun = 5° and 0° respectively; see Fig. 3) and the observed arcs (see Fig. 1) shows a general similarity, but there is a striking discrepancy in the angle enclosed by the two wings of the arcs. This discrepancy, which amounts to about 30° and 15° when the altitude of the sun is assumed to be 5° and 0° respectively, may be explained by the well-known systematic error occurring when geometric dimensions in the sky are estimated visually without adequate instruments. Observational errors may account for the remaining differences in the dimensions of the observed and calculated arcs, particularly in the distance between the normal and abnormal arcs. It may also be noted that the calculation of the arcs was based on geometric optics and disregarded the effect of diffraction, which may be significant.

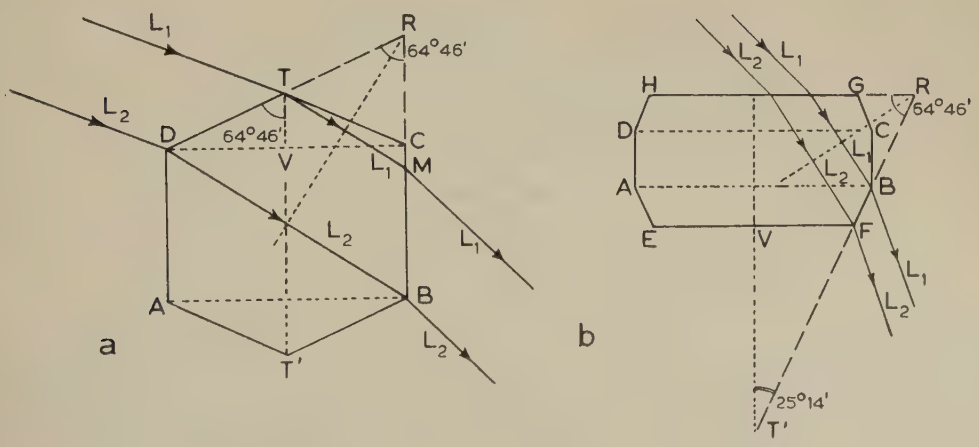

Figure 4 (a) and (b). The crystal forms discussed.

4. The two crystal forms

1a. Inclination 64°46′ (Fig. 4a). Cross-section AT′BCTD of the crystal perpendicular to the refracting edge R, principal axis TT′, face of entry TD, face of exit CB, inclination of the pyramidal face, angle DTV, = 64°46′, refracting angle at R the same, limiting rays L_1 and L_2, passing through the crystal at minimum of deviation. If r is the length of the edge of the regular hexagonal section of the prismatic part, then

$AB = r\,3^{\frac{1}{2}} = 1{\cdot}73\,r$

$AD = 1{\cdot}09\,r$ (the height of the prismatic part)

$TV = 0{\cdot}41\,r$ (the height of the pyramid)

$CM = 0{\cdot}13\,r$ (M is the point where the upper passing ray meets the prismatic face).

The shape of this crystal is extremely unfavourable, since it deviates considerably from the required plate. When the height CB of the prismatic part is diminished the plate form improves, but if it is reduced to the length CM no light can pass through the crystal. It is evident that no light can pass when the pyramid is truncated by a base plane.

1b. Inclination 25°14′ (Fig. 4b). Cross-section AEFBCGHD, inclination of the pyramidal face, angle VT′F, = 25°14′, refracting angle HRF = 64°46′, face of entry HG, of exit BF, limiting rays L_1 and L_2.

The refraction is now nearly independent of the height of the prismatic part. The crystal completely fulfils the conditions of the plate, and it can be concluded that it was this crystal form in which the abnormal tangent arc developed.

Only once before, as far as we know, was a tangent arc of an uncommon halo observed. It was the upper tangent arc of the halo of 11½°, observed by Ir J. G. Lepper at Aerdenhout (near Haarlem, Netherlands) on 15 April 1933 (*R. Neth. Met. Inst. 1936*). In this case too, the refraction occurs between the base and the opposite pyramidal face, the refracting angle being 25°15′ and the inclination of the pyramidal face 54°44′, just the angle measured by Clarke.

In both phenomena we are concerned with refraction in similar plate-shaped ice crystals, which differ only in the inclinations of the pyramidal faces.

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Roy. Neth. Met. Inst.	1936	*Onweders, 1933*, **54**, p. 46.
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Graphs of cumulative residuals

By E. B. KRAUS

*The Snowy Mountains Hydro-Electric Authority, Cooma, N.S.W., Australia**

(Manuscript received 26 September 1955)

Summary

The particular uses of cumulative residuals are demonstrated by two plots of annual Nile discharge at Aswan. It is shown that the mean regime of the Nile changed abruptly in 1898 in conformance with secular rainfall changes throughout most of the tropics. No trend is apparent in the two periods before and after 1898.

* This note has been prepared during the tempory secondment of the author to the UN/WMO Technical Assistance Mission for Desert Locust Control, Nairobi.

In two recent papers (Kraus 1955a, 1955b), cumulative residuals, that is cumulative deviation from a mean, were used to illustrate changes in the rainfall regime. The usefulness of the procedure was queried by several correspondents who failed to see its advantages as compared with conventional graphs of individual values, or of 30-year running averages. The particular advantages of cumulative residuals are perhaps best demonstrated by two plots of the same quantity. Fig. 1 gives the values of the annual Nile discharge at Aswan as quoted by Hurst and collaborators (1946). Fig. 2, based on the same data, represents a residual mass curve.

Discharge from the Nile has shown a markedly decreasing trend. This is immediately suggested by a simple inspection of Fig. 1. The linear regression equation is $y = -5{\cdot}217\,x + 9024$ where y is the discharge in 10^8 m^3/year and x is the year A.D. The trend shown by this equation is statistically significant at the 1 : 1,000 level. With some further labour, it would have been possible to compute a non-linear regression curve. It would be difficult or unjustifiable, however, to draw any further conclusions as to the character of the time series from a simple inspection of a graph such as Fig. 1.

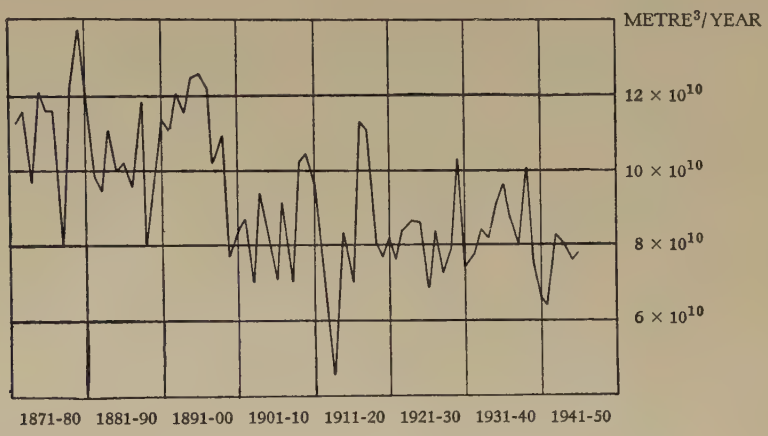

Figure 1. Annual discharge of the Nile at Aswan.

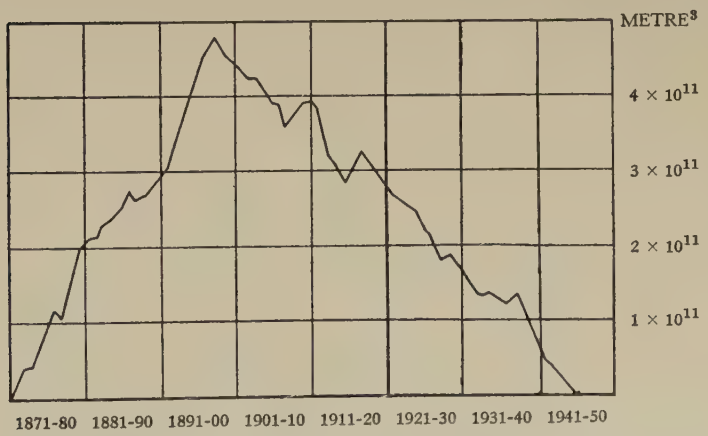

Figure 2. Cumulative residuals of annual Nile discharge at Aswan.

The graph of Fig. 2 being convex upwards shows the diminishing trend of Nile-flows even more readily than Fig. 1. In addition, it shows that the Nile regime appears to have changed abruptly in 1898. The 1871-1945 mean of the discharge was $9{\cdot}297 \times 10^{10}$ m^3/year. The difference between that value and the mean for any other period, say the years m to $m + n$, is given immediately by the formula $(Y_{m+n} - Y_n)/m$ where the Y's are the values of the

cumulative residuals. In the present case $Y_{1898} = 4{\cdot}761 \times 10^{11}$ m^3. The mean discharge for the years 1871-1898 was therefore :

$$\bar{y}_{1871-98} = (4{\cdot}761 \times 10^{11} - 0{\cdot}0)/28 + 9{\cdot}297 \times 10^{10} = 10{\cdot}997 \times 10^{10} \text{ m}^3/\text{year}$$

whilst the mean discharge :

$$\bar{y}_{1899-1945} = 8{\cdot}284 \times 10^{10} \text{ m}^3/\text{year}$$

Fig. 2 also immediately shows that there can have been no significant trends in the two periods before and after 1898. What happened appears to have been a discontinuous change from one type of river regime to another.

In the case of the Nile, the decrease between the two means amounts to 33 per cent. The decrease has been so large that it can be discerned even in Fig. 1. Normally, however, it is extremely difficult on a graph of individual values to discern long-term changes in a time series with large short-term variations. On the other hand, the graph of cumulative residuals will not only demonstrate in a convincing manner the existence of a trend, but also its value for any given period and the time of changes in the trend. This is partly owing to the fact that it is easier for the eye to discern changes in average slope than changes in mean level. In addition, graphs of cumulative residuals tend to accentuate changes in a time series whilst running averages or orthodox statistical analysis tend to smooth them out. The computation of cumulative residuals requires less calculating labour than the computation of running averages or of a linear regression.

The discontinuity in the Nile regime is so drastic and pronounced that it suggests a mechanical change in the river bed or the gauging controls at Aswan. The change occurred, however, in the same year, 1898, in which changes of the rainfall regime can be observed in the records of most tropical stations at places as far apart as Northern Australia and Central America (Kraus 1955a, 1955b), It represents therefore, very probably, an actual discontinuity of the Nile regime. If the change in the Nile-flow appears to be more clear-cut than that of most rainfall stations, it is due to the integrating effect of a river catchment, which will be less affected by the vagaries of particular storms than are individual rainfall stations.

In the case of both rainfall or stream-flow, graphs of cumulative residuals will give immediately the excess or deficit of available water. This is one reason why this type of graph is much used for example, by hydrological engineers in the design and operation of storages. Such an immediate physical interpretation is not available when these graphs are used to illustrate changes of physical variables which have not the dimension of a mass flow. The graphs retain, however, even in that case their property of showing very clearly changes in time series. They have been found a useful tool in the investigation of secular pressure and temperature changes.

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Correspondence

551.575.1 : 628.8

Low-temperature fog at the Edmonton Airport as influenced by moisture from the combustion of natural gas

By H. Appleman

As Mr. Robertson indicates in his paper (1955), it is possible for the combustion of hydrocarbon fuels to liberate sufficient water vapour to saturate the environment and produce a fog. In his basic equation, however, Robertson neglects the heat of combustion. While the added water vapour tends to raise the relative humidity of the environment, the added heat tends to lower it. The resulting relative humidity may be either greater or less than the original value, depending on the initial pressure, temperature, and relative humidity of the air. This is the reason why, above a certain temperature, the combustion of hydrocarbon fuels can be used to dissipate fogs (e.g., the Fido method), while at lower temperatures such combustion actually produces fogs.

I studied this phenomenon in connection with research into the formation of condensation trails (1953a) and ice fogs (1953b) resulting from the combustion of kerosene and gasoline. At a pressure of 1,000 mb, the critical temperatures for saturation to occur ranged from — 20°F to — 39°F for initial relative humidities ranging from 100 to 0 per cent with respect to water. In Fig. 3, Robertson shows the occurrence of combustion saturation at temperatures above — 20°F for an initial relative humidity of 95 per cent, and at unlimited temperatures for initially saturated conditions. Methane gas has a slightly higher water vapour to heat ratio in its exhaust than does either gasoline or fuel oil. Hence the critical temperatures for combustion saturation would be slightly higher. However, in no case would the combustion of natural gas result in saturation at temperatures much above — 20°F.

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Headquarters Air Weather Service,
Andrews Air Force Base,
Washington, D.C.
31 May 1955.

Reply

By G. W. Robertson

In reply to Mr. Appleman's valuable comments I have to admit, unfortunately, that quantitative consideration was given neither to heat produced by combustion, nor that lost by radiation, which is equally important.

Since the observing point at which temperatures were measured was at the airport within the zone of admixture of combustion gases and environment air, it was felt that the measured temperature reflected the influence of the heat of combustion as well as of radiational cooling.

It must be remembered, furthermore, that less than 30 per cent of the heat from combustion in a dwelling-furnace is ejected from the chimney. The greater percentage of the heat of combustion is slowly dissipated through the walls and roof of the dwelling where a large percentage may be lost by radiation without actually increasing the temperature of the surrounding air.

Dept. of Agriculture,
Experimental Farms Service,
Ottawa, Ontario.
23 September 1955.

551.508.765 : 551.574.1 : 551.578.11

The coalescence of small water drops

By Doyne Sartor

The coalescence efficiencies reported by Telford, Thorndike, and Bowen (1955) are surprisingly high. Their explanation of the collision mechanism effective in producing high coalescence efficiencies agrees, in general, except for degree, with that reported by Sartor (1954) for droplets with Reynolds numbers in the vicinity of 0·1. Trajectories of two colliding droplets taken from slow-motion movies are reproduced in Fig. 3 of the latter paper and show the acceleration of the larger droplet toward the smaller one even at these small Reynolds numbers. In this case, however, the acceleration did not continue to the point of collision, but deceleration began when the droplets were two or three radii apart as would be expected from an examination of the streamlines for small Reynolds numbers. The force of collision was not sufficiently great to effect coalescence unless an attractive force was provided by superimposing an electric field. Although these observations were for Reynolds numbers much less than those of the Telford, Thorndike, and Bowen experiments, it is difficult to understand the increasing acceleration right up to the point of collision and the subsequent ready coalescence they observed.

It would be of interest to know the shape of the velocity profile in the channel where their measurements were made. They report that, ' The velocity profile is extremely flat, as can be seen when turbulence is allowed to build up, and a drop wanders about horizontally. The boundary layer becomes apparent only within about ½ inch of the walls.' A flat velocity profile is to be expected when ' turbulence is allowed to build up,' but when the turbulence decreases, as was apparently the case under their working conditions, the profile becomes much sharper with a velocity maximum near the centre. Where a sharper velocity profile exists the drops would tend to accumulate near the centre of the stream where the velocity is greatest and remain in a constricted column in the manner reported by the authors. Under these conditions, collision efficiencies are likely to be over-estimated.

The shielding of the working area from electrical fields, such as those of the 250-w landing lamp, the 3·5 kw carbon arc light, the charging wire, the blower motor, and the clearing field, is also a matter of some concern. The writer has observed marked mutual attraction and coalescence between uncharged water drops of size similar to those employed by Telford, Thorndike, and Bowen only when an electrical field was applied. These drops were falling freely through mineral oil in a plexiglass container, an arrangement not significantly different electrically from their experiment. The effect of an electric field in producing mutual attraction and coalescence under these circumstances can be observed qualitatively by forming a number of small drops of water in a glass or plastic container filled with mineral oil and holding near the container a hard rubber comb which has been charged by passing it through the hair a few times.

Experience has shown that it is difficult to shield experiments like these from electrical fields in the laboratory. It must be equally difficult to obtain a flat velocity profile under laminar flow conditions. When electrical fields are present or the velocity profile is peaked, the coalescence efficiencies are likely to be larger than if these conditions are absent. Thus it is important to evaluate the results in the light of quantitative measurements of electrical fields and velocity profiles existing in the working section.

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Technical Support Branch,
2D Weather Wing, (MATS),
c/o Weather Detachment 18-29,
Rhein-Main Flughafen,
Frankfurt a.M., Germany.
13 October 1955.

551.554

Vertical wind profiles under non-adiabatic conditions

By Richmond W. Longley

Deacon (1949, 1953) introduced to meteorologists the possibility that the wind profile near the ground could be described by the equation :

$$\frac{du}{dz} = az^{-\beta} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In this equation, u is the horizontal wind speed at a height z, and a and β are constants in any given situation. Deacon showed that the parameter β was a function of the stability and related to the Richardson number.

Since its introduction, the parameter β has been evaluated by a number of workers who have assumed that it has given them a measure of stability. Most of these have not questioned the accuracy of their answers, but Rider (1954) concluded that Eq. (1) was satisfactory under unstable, but not under stable, conditions. Davidson (1952) developed graphs and nomograms so that, knowing β and z_0, the roughness parameter, he could determine the concentration of a gas downwind from a source. An investigation into the evaluation of β seemed warranted.

Integration of Eq. (1) leads to

$$u = \frac{a}{1-\beta}\{z^{1-\beta} - z_0{}^{1-\beta}\} \quad . \quad . \quad . \quad . \quad (2)$$

where z_0, a constant of integration, is the value of z when $u = 0$. If it is assumed that z_0 as determined under adiabatic conditions is unchanged with non-adiabatic conditions, the values of a and β may be obtained from observations at two levels. If this assumption is not made, the parameters may be determined when u is observed at three levels. In general, because of the exponential form of Eq. (2), the evaluation is not easy. However, if the three levels z_1, z_2 and z_3 are related by

$$z_3 = rz_2 = r^2 z_1$$

where r is any ratio, Eq. (2) may be used to obtain the relationship

$$\beta = 1 - \frac{\log \dfrac{u_3 - u_2}{u_2 - u_1}}{\log r} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where u_i is the wind at z_i.

If Deacon's (1953) data are used with this equation to recompute β using different groupings of the levels at which measurements were taken, the range of the differences found between Deacon's values of β and those computed using the winds at 0·5, 1 and 2 m is from 0 to 0·22 with a mean of the absolute values 0·10. Using winds at 1, 2 and 4 m, the corresponding mean is the same, i.e., 0·10. For the levels 2, 4 and 8 m, the mean is 0·20; and for the levels 0·5, 2 and 8 m, 0·12. Since 13 out of 18 of Deacon's values of β lie in a range of 0·20, a possible error of 0·10 does not permit different types of stability to be distinguished clearly.

Eq. (3) was also used to compute β using hourly mean data from Halstead (1952) and 7-min mean data from a number of trials at Suffield Experimental Station. The values computed using the Suffield data were very erratic, and even those using hourly mean data varied for the same hour. During the daylight hours, the range of the four values of β for the same hour lay between 0·1 and 0·2. During inversion conditions, the ranges were much greater.

Rider (1954) presented the results of 51 sets of observations of winds taken at 8 different levels. These data were used to evaluate the parameter in another way. Straight lines on semi-logarithmic paper were drawn through Rider's computed value of z_0 (0·30 cm) and $u = 0$, and through (u, 37·5 cm) and the deviations from these lines determined for the other levels.

Curves of the form given by Eq. (1) were computed to pass through (0, 0·30 cm), (u, 37·5 cm) and (u, 150 cm), and again the deviations at the remaining levels computed. Table 1 gives a comparison between the two sets of results.

TABLE 1. COMPARISON OF DEVIATIONS FROM A STRAIGHT LINE (ON SEMI-LOG PAPER) AND A CURVE DETERMINED BY THE VALUE OF β

Height (cm)	No. of observations in which the better estimate was made by		Sum of absolute value of deviation from	
	line	curve	line	curve
200	6	34	775	290
100	13	29	342	239
75	19	21	230	209
50	21	19	218	208
25	31	12	364	431
15	28	15	545	678

The method of computation was such that the deviations were 0 for the straight lines at 37·5 cm, and 0 at 37·5 and 150 cm for the curves. It will be noted that for the top two levels the curves give the better fit; at 75 and 50 cm, there was little difference; near the surface the straight lines were better.

Near the surface, Eq. (1) represents the change in wind profiles under changing stability conditions. Nevertheless, it is impossible to calculate β with sufficient accuracy to warrant its use in any practical problem.

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Suffield Experimental Station,
Ralston, Alberta,
Canada.

23 June 1955.

REPLY

By E. L. DEACON

It is not surprising to me that Mr. Longley finds the wind-profile parameter β hard to measure. This was rather obvious from the beginning and I have never been so optimistic as to suppose that in practical problems it would be profitable to attempt the routine measurement of β in the field. When the attempt was made during the last war to equip some field units for the measurement of the 2 m : 1 m, wind ratio (R), for use in Sutton's formulations, it was soon found that the practical difficulties of exposure and non-uniformity of surface were too great and that a much more satisfactory approach was by establishing the relationship between R (over a standard surface) and such factors as time of day, season, latitude, state of sky, and wind velocity. Simple tables then made the estimation of R a rapid and reliable process.

In an unpublished report which nevertheless should be available to Mr. Longley, I suggested in 1947 that ultimately the most satisfactory way of obtaining β would be by establishing the relationship between it and the Richardson number for an appropriate level or layer. With this established it would be relatively easy to measure (or estimate) the Richardson number with sufficient accuracy for β to be found within close limits. I went on to say : ' The task of establishing the β : Ri relationship will, however, be no easy one and will require a long series of very accurate temperature and wind gradient measurements over very uniform country.' It now seems that in this type of problem, meteorologists will have to face the necessity for the replication of measurements in a similar manner to that long since found necessary in agricultural science. This is a great but apparently unavoidable nuisance.

Whether or not the β formulation stands the test of time, it is now an established fact that the eddy diffusivity in the surface layers increases more rapidly with height than linearly under unstable conditions, and less rapidly in a stable stratification. Therefore any acceptable treatment of diffusion problems in these layers *must*, in some way, take account of this and in so doing encounter similar difficulties to those associated with the measurement of β.

Division of Meteorological Physics,
C.S.I.R.O.,
Aspendale, Victoria,
Australia.
29 October 1955.

551.594.25

The charging of rain by splashing

By Sir George Simpson, F.R.S.

May I support Dr. J. A. Chalmers (1955) in his criticism of L. G. Smith's (1955) theory that the usual negative potential gradient and positive rain-current which are found in quiet rainfall are to be accounted for by the splashing of the rain as it reaches the ground ? Chalmers gives four examples in which Smith's theory is in disagreement with observations; I should like to add one more :

On the voyage of the *Terra Nova* to the Antarctic in 1911, Simpson and Wright (1911) made many observations of the potential gradient in mid-ocean. The following is quoted from their report :

> The effect of rain (on the potential gradient) was found to be the same on the sea as on the land. As the rain approached, the potential gradient generally decreased, and during one period of steady rain, negative potential gradient was noted. When, however, the rain was associated with a squall the potential gradient took on large values, which changed rapidly from one sign to the other just as it does in a thunderstorm on land.

It is well known that when drops of distilled water splash on a surface of salt water, a very small negative charge remains with the salt water and the corresponding positive charge goes into the air (Simpson 1909). Thus the electrification due to the splashing of rain drops at the surface of the sea is not only less than on the land, but it is of the opposite sign, yet the potential gradient is the same over the sea as over the land. It is difficult to see how Smith's theory is to be reconciled with these observations.

I would like to take this opportunity to plead for more work on atmospheric electricity over the oceans, where little has been done since the loss of the *Carnegie* in 1929. It seems to me that many of the problems which have given rise to unaccepted theories in atmospheric electricity during recent years could be settled by observations at sea. For example, what is the part played by the point-discharge current in the electricity carried down by rain and the charge remaining in the clouds ? A comparison of the results obtained on land, where the point-discharge current appears to be all-important, with similar observations at sea, where there is no point-discharge, should be decisive.

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5 Mansfield House,
Manor Fields,
Putney, London, S.W.15.
12 November 1955.

551.511 : 551.557 : 551.547

Geostrophic and ageostrophic wind analysis

By F. B. Smith

With reference to my paper which recently appeared in the *Q.J.* (**81**, p. 403, 1955) I feel there is some doubt as to how the measurements were taken. The method was briefly as follows. The data used were the values of height at each of the nine pressure levels at the stations cited in § 4 (whenever these were available). These values were plotted on a map of the British Isles (scale 100 km/in.) and the contours drawn for each pressure level. Since a contour is also an isobar in a horizontal plane, the difference in the directions of the contour and the wind (wind data taken from official records), the wind speed and the measured radius of curvature of the contours taken at the radiosonde stations enabled the values in Table 1 to be calculated. The means of a relatively large number of such readings were taken to give sufficiently accurate values.

Manchester University.
19 September 1955.

551.508.77

Investigation into the accuracy of four types of rainfall recorder

By W. A. LEWIS and L. H. WATKINS

(Read 19 October 1955. See *Q.J.*, **81**, p. 449)

Mr. J. R. Bibby : Comparing the Road Research Laboratory recorder with other types, there seem to be two possible sources of trouble : (a) pieces of solid matter sticking in the needle valve, (b) uncertain action of the syphon. It would be interesting to know whether any such trouble was found.

Regarding the Meteorological Office rate-of-rainfall recorder being frequently out of adjustment, this instrument was largely re-designed about two years ago, and is now much more reliable.

Mr. E. Gold : I should like to ask what was done to maintain the water level in the intermediate vessel during a period of dry weather, when perhaps all the water would evaporate. Also, what is the cost of the instrument ?

Mr. F. H. Ludlam : It seems that the principal difficulty in evaluating the rate of intense rainfalls from the record of the conventional automatic gauge is in determining accurately their duration; would it not be possible to overcome this by some arrangement for making frequent time-marks by a short *horizontal* movement of the pen ?

Mr. E. J. Ball : I would like to know what happens when rates of rainfall greater than the highest recordable (namely, 5 in./hr) occur. Often in summer storms, much greater rates of rainfall occur than in heavy winter falls. Perhaps the actual area covered by a ' downpour ' which lasted only a few seconds, was so small that, taking these two points into consideration, the record, here, at least, is only of academic interest. Could some form of eddies round the gauge itself ' concentrate ' the rain into a shape which would then fall into the gauge ? Has this question been proven to have the answer ' no ' ?

Mr. L. H. Watkins (*replying on behalf of both authors, partly communicated*) : Mr. Bibby has pointed out the two most likely sources of trouble with the Road Research Laboratory recorder, but I can assure him that with regular servicing very little trouble is experienced. A small trap at the bottom of the collecting funnel and an efficient filter between the funnel and the instrument remove the solid matter which is large enough to stick in the orifice, and weekly cleaning of the trap and filter, flushing the instrument through with water at the same time, has been found sufficient to maintain satisfactory operation. The system is very reliable in its operation, although it took a considerable time to develop a syphon which would both prime instantly during low rainfall and also break instantly during heavy rainfall without any tilting mechanism which would have added to the complexity of the instrument. The final design incorporated a slight flattening of the syphon tube at its highest part to assist priming, and a large rectangular air space between the syphon tube and the float chamber which allowed a large volume of air to enter the tube, thus breaking the syphon cleanly.

We are interested to learn that the Meteorological Office rate-of-rainfall recorder has been modified and is now more reliable.

The intermediate vessel referred to by Mr. Gold is presumably the trap mentioned above. This trap is allowed to become dry when there is no rain, but since its capacity is very small (about 20 cm^3) less than 0·001 in. of rain is required to fill it.

The Road Research Laboratory instrument is now being manufactured commercially, the cost per instrument ranging from £254 to £271, depending on the number ordered.

In reply to Mr. Ludlam, the principal difficulty which we have experienced with the conventional record of total rainfall is not simply that of determining the duration of a given rate of rainfall. Our problems have been :

(1) to determine precisely the time at which the maximum rate of rainfall occurred and its duration. This can be instantly determined from the record of rate of rainfall.

(2) to determine accurately the rate of rainfall minute by minute. This is most easily done by increasing sufficiently the speed of the chart of the total-rainfall recorder, although the record of rate of rainfall is also useful in this connection when correcting for the loss during the syphoning period, which is of course important when considering such short duration.

The horizontal movements of the pen on a conventional total-rainfall recorder suggested by Mr. Ludlam would not, I feel, help to overcome these difficulties, and might in fact only make the record rather less clear than it is at present.

Mr. Ball is quite correct in saying that greater rates of rainfall are recorded during the summer than during the winter. In this country when the rate of rainfall exceeds 5 in./hr, however, it generally does so for only very short periods and the area covered is probably small. Thus as Mr. Ball suggests, rates of rainfall of over 5 in./hr are only of academic interest. The larger the catchment area, of course, the less the importance of these very high, short duration rates of rainfall.

The answer to Mr. Ball's last question concerning the possible concentration of the rain by wind eddies around the Road Research Laboratory's instrument is, I think, provided by the fact that our field investigations with the four different instruments revealed no marked difference in the total rainfall recorded by them during a storm. If this effect place takes with the Road Research Laboratory instrument it presumably therefore also takes place to the same extent with all the others. In view of the very different dimensions of the collecting funnels of the instruments this is unlikely to be the case.

551.578.11

The salinity of rainfall as a function of drop size

By J. S. TURNER

(Read 19 October 1955. See *Q.J.*, **81**, p. 418)

Dr. R. M. Goody : It has been shown by Swedish workers that the ratio Cl/Na in rainwater can vary from zero to about four. For sodium chloride the ratio is 1·5. The sodium concentration in rainwater is therefore a very poor indicator of the amount of sodium chloride which has entered the rainwater. This must have a considerable influence upon the conclusions which Mr. Turner draws with respect to concentrations of sodium chloride, although it is not obvious how the interpretation of his results can be improved. It would certainly help if chlorine concentrations could be measured as well as sodium concentrations, although this may present great analytical difficulties with the small amounts of water which are available.

Mr. L. C. W. Bonacina : I should like to know to what extent experiments of this kind could be made during heavy gales at increasing distances from the coast.

In severe tempests salt spray mixed with the finer raindrops will be driven fifty or more miles inland, searing the foliage of trees at all exposed points.

Dr. R. C. Sutcliffe (*President*) : Do the experiments allow of an estimate of the sizes and numbers of the large nuclei which are inferred to be present ?

Mr. J. S. Turner (*in reply*) : I agree with Dr. Goody that the measurement of the concentration of only one element will not necessarily indicate the total amount of sodium chloride present in rain. Sodium was chosen primarily because of its convenience when working with small samples, but it seemed reasonable to suppose that near the coast, and especially on an island far from any sources of nuclei other than the sea, the mean Cl/Na ratio would be not very different from that for sea salt.

In the present context, however, the important point is not the absolute value of the concentrations, but the ratio for raindrops of different sizes. The interpretation suggested will remain valid provided the ratio of Na to the total weight of salts present is the same for nuclei of all sizes over periods of a few hours; and even if this is not so, there is still some selective mechanism to be explained.

In reply to Mr. Bonacina I would say that we did in the early stages of the experiments set up a spectrograph near the sea, and measured very high salt concentrations during strong winds. Since collection of salt below the cloud was regarded only as an obscuring influence for our purpose, systematic measurements of this sort were not continued.

Answering Dr. Sutcliffe, I would be cautious about deducing nuclei sizes and numbers from these results. This could be done simply if it were permissible to assume that only one nucleus was present in each raindrop, but I suggest there is reason to believe that raindrops can contain several large nuclei.

551.509.67 : 551.577.21 (54)

The cloud-seeding trials in the central Punjab, July-September 1954

By E. M. FOURNIER D'ALBE, A. M. A. LATEEF, S. I. RASOOL and I. H. ZAIDI

(Read 19 October 1955. Presented by F. H. Ludlam. See *Q.J.*, **81**, p. 574)

Mr. B. J. Mason : I should like to congratulate the authors on their courage and ingenuity in undertaking these experiments with such limited facilities. The following remarks are not intended as criticism of the authors, who have been careful not to claim too much for the efficiency of their experiments, but rather to draw attention to the difficulty of drawing definite conclusions from such short-period trials conducted on a purely statistical basis.

In the first place, I doubt whether an appreciable increase in rainfall from these clouds could be brought about by increasing the concentration of large hygroscopic nuclei from say $5/m^3$ to $10/m^3$ unless a water-drop chain reaction were set up in the cloud. Were there clouds of sufficient depth and vigour to make this likely ?

With regard to the rainfall statistics, the excess rainfall in the target area A relative to that in control area B, as deduced from the measured rainfall during the trial period expressed as a fraction of the historical average, is confined mainly to the narrow 20-km strip enclosed by arcs of radii 80 km and 100 km drawn with Lahore as centre. Thus, the average values of the figures shown on the chart for the 8 long-period stations in this narrow strip is 0·60 compared with a corresponding value of 0·42 for the whole of the B control area, *but* the average value for the *whole of the remainder of the target area* is only 0·40. Bearing in mind that the concentration of the seeding agent may have been very low at this extreme downwind-edge of the target area, the suggestion that the higher rainfall there may have been produced by the seeding is not very convincing. One is more impressed by the abnormally low rainfall occurring in area C than by the possibility of increased rainfall in A.

One regrets that the statistics could not have been supplemented with some observational data on the clouds themselves. A study of the cloud development before, during and after seeding, observations of the onset of precipitation in relation to the estimated time of nucleation, etc., would have been most desirable. In this kind of work, a favourable statistical result carries so much more conviction if it can be supported by some physical evidence, and I should like to recommend strongly that such observations be attempted in any future extension of this work.

I think that it can be said that the results of these experiments are, at any rate, not discouraging and I hope that they will be continued.

Dr. R. S. Scorer : Mr. Mason argues that at a distance of more than 50 km from the seeding point no concentrations of salt nuclei could be produced which would significantly affect the rainfall, and that therefore the rainfall observed at greater distances could not be due to the seeding. However, the rainfall figures are sufficiently impressive, surely, to cast doubt on the basis of Mr. Mason's argument, which is, after all, only theoretical !

Mr. E. Gold : According to Eliot's *Climatological Atlas of India* the wind at Lahore in July and August is mainly easterly, the isobars being approximately SE to NW. I suggest that the drift of the salt particles – or of those effective in the clouds – at some kilometres from the source would be dependent on the upper wind, probably from the SE. If so the boundaries of the sector A should be curved northwards, not straight lines.

Possibly the rainfall in the strip 100 km from Lahore is affected by orographic influences : there is high ground in that direction, though rather further from Lahore.

The results appear to me to warrant further trials – they do indicate an effect of the salt and I think Pakistan is to be warmly congratulated on making these experiments and at so small an expense.

Dr. G. A. H. Elton : It is, of course, impossible to draw any definite conclusions concerning the effect of the seeding from a study of the rainfall statistics of a single year. Even

if results of the kind given in the paper were obtained for several years running, it would not be possible to rule out entirely the possibility that this was due to chance. For this reason, it would seem to be of the utmost importance to check experimentally whether the operation of the seeding apparatus does in fact lead to an appreciable increase of the number of condensation nuclei at distances up to 100 km. If possible, measurements should be made at cloud level, but even if this cannot be done, measurements made at the ground would be better than none at all.

Mr. B. C. V. Oddie : It is perhaps somewhat barren to discuss the precise significance of these results when no absolute significance is claimed for them. At the same time, if the authorities who are paying for this work do not consider that these results justify continuing, they must be very hard to please. It is to be hoped that more detailed attention will be given in future to the actual tracks of the salt particles. If one merely assumes that they are in a wide and not very rigidly defined sector, it is usually possible to find some area in that sector which has a high rainfall on a given occasion, and it is easy to assume this is due to the seeding. It would, however, be better to relate the result to the wind on that particular occasion.

Dr. R. C. Sutcliffe (*President*) : The angle of the sector chosen as representative of the seeded area is about 55°. The plume from a point source on any one occasion is hardly likely to exceed 10°, so that any part of the area may be seeded only one-fifth or less of the time. On this basis the argument for the positive effect is considerably strengthened but one remains uneasy about the chosen radius of the sector. Had this been a little smaller the positive area would not have been included. However, we must congratulate the authors and authorities concerned for their originality and enterprise in doing something really useful with very limited resources and we must look forward to further results.

Mr. F. H. Ludlam (*in reply*) : In the discussion on this paper doubts have been raised about the possibility of salt seeding having any significant effect at a distance of 100 km, and about the validity of the rainfall analysis. However, present views on the mechanism of salt seeding and shower formation are insecurely founded and can be used either to support or attack the interpretation of the results. The experiments have been analysed on a seasonal basis, and several seasons of such experiments will of course be necessary to provide confidence in the reality of the results.

I think the authors have done fully as well as is possible with their resources. Advance in the artificial modification of weather depends essentially upon knowledge which can be gained only with the help of expensive facilities, such as special aircraft and radar. In the discussion the President asked whether salt seeding of the kind described should in the view of members be undertaken in this country, where facilities of the most advanced kind are readily available. I think these should rather be fully occupied with those fundamental investigations in cloud physics which are so pressingly needed for the improved design, execution and evaluation of experiments in weather control.

Dr. E. M. Fournier d'Albe (*in reply on behalf of the authors, communicated*) : As Mr. Mason pointed out, the highest rainfall in the 'target' area A was recorded in the strip between the 80 and 100 km radii from Lahore. The distance downwind of the seeding station at which the concentration of salt particles in the plume fell below the background level undoubtedly varied greatly from day to day. We agree with the President that the fixing of the limit of the target and control areas at 100 km from Lahore, though it has some theoretical justification, is in fact rather arbitrary. However, even if we deliberately exclude from the target area the region of relatively high rainfall between 80 and 100 km from Lahore, the final result is not greatly affected. We now have six stations with 14-yr records in area A, and the average rainfall at these stations in 1954 was 0·40 of normal. This leads to a value of $\log (A/\sqrt{BC})$ equal to + 0·15, which was only once equalled and once exceeded in the 14-yr period. Of the three stations with 42-yr records in area A, we now retain two, and the new value of $\log (A/\sqrt{BC})$ for comparison with the 42-yr series is + 0·24. This value was only exceeded once in the period 1901-42.

As Mr. Gold remarked, the wind in the monsoon-air masses tends to veer slightly with altitude and we cannot be certain that some of our salt did not get into area B. We were indeed acutely conscious of our inability to supplement the statistical data with physical observations. We did in fact prepare charts of daily rainfall in which the times of commencement and end of precipitation are shown, together with surface and upper wind directions at five pilot-balloon observatories. Some of these daily charts showed very interesting rainfall patterns, but we felt unable to draw any conclusions from them in the absence of sufficiently detailed knowledge of the surface and upper winds and their fluctuations throughout the day.

Admittedly the main weakness of the experiment lay in the absence of any direct observation of the trajectory and diffusion of the plumes of salt particles. Dr. Elton and Mr. Oddie have suggested sampling the air at the ground and from aircraft at points downwind of the seeding station. This would indeed be ideal but would require tremendous resources. To obtain a sufficiently detailed picture of what is happening one would require, I fear, not one but a fleet of aircraft and a dense network of ground observatories. A possible answer to this problem may lie in the chemical analysis of the rainwater. For instance, if anomalous values of the sodium or chloride content were found to coincide with anomalous rainfall amounts, that would provide a strong positive indication, though still no proof, of the activity of the dispersed salt particles.

Returning to a point raised by Mr. Mason, the observed intensities of shower rainfall in the Punjab do suggest that drop multiplication by fragmentation is a frequent occurrence. If conditions favourable to the water-drop chain reaction persist in a cumulus cell for more than 10-15 min it is likely that by the end of that time the rate of production of raindrops by fragmentation will greatly exceed the rate of production by condensation and coalescence on incoming large nuclei. In this case one may wonder whether the introduction of additional large nuclei can have any appreciable effect. However, the life of cumulus cells is rarely long enough for the chain reaction to develop to this extent, and the time during which fragmentation may continue appears to be generally much shorter. In this case the rate of formation of raindrops will depend on the number of primary fragmentations and thus on the number of drops growing by condensation and coalescence on large nuclei.

With regard to the question raised by the President as to whether seeding experiments with salt particles might be attempted in England, I would hazard the guess that the natural concentration of giant hygroscopic nuclei in this country is usually very high. However, there are parts of the British Commonwealth and Colonies where conditions are likely to be much more favourable.

Ordinary Meeting, 19 October 1955

The following candidates were declared to have been elected by the Council during the summer recess :

Fellows

Bhat, Umesh Ramkrishna, M.Sc., c/o Div. Operations Mgr., Air India International, London Airport, Hounslow, Middx.
Dack, John Robert William, Dept. of Civil Aviation, P.O. Box 87, Accra, Gold Coast, W. Africa.
Elton, Dr. George Alfred Hugh, 101 Howard House, Dolphin Square, London, S.W.1.
Graham, Robert Campbell, 315 Bloor Street West, Toronto 5, Ontario, Canada (*transfer from Associate Membership*).
Irving-Bell, Ronald Johnston, M.R.C.S., L.R.C.P., D.P.H., 5a Oakfield Road, Bristol 8, Somerset.
Newman, Bernard William, B.Sc., Weather Bureau, Brisbane, Queensland, Australia (*transfer from Associate Membership*).
Pedlow, Robert Henry, B.Sc., 21 North Drive, Shortstown, Bedford.
Rowntree, Norman Andrew Forster, 43 Kingswood Road, Tadworth, Surrey.
Stanley-Clarke, Bryan Graeme, B.Sc., Rossearn, Comrie, Perthshire, Scotland.
Stanton, Walter James, 'York Lodge,' 216 Park Crescent, Erith, Kent.
Wilcox, Instr. Lt. Kenneth Arthur, B.Sc., R.N., H.M.S. Daedalus, R.N.A.S. Lee-on-Solent, Hampshire.

Foreign Members

Haltiner, George J., 1134 Alta Mesa Road, Monterey, California, U.S.A.
Koroleff, Nils Holger, M.A., Stationsvägen 10, Grankulla, Finland.
Lake, Herman, M.S., 9 Appletree Lane, Lexington, 73, Mass., U.S.A.
McCormick, Robert A., Kew Observatory, Richmond, Surrey.
Nagel, Johann Friedrich, Dr.Rer.Nat., 347 Leydsst, Pretoria, S. Africa.
Osborn, Byrle Jacob, Ballyhenry, Ashford, Co. Wicklow, Eire.

Associate Members

Spillane, Kevin Thomas, B.Sc., c/o Weather Bureau, Brisbane, Queensland, Australia.
Whittingham, Herbert Edwin, B.Sc., c/o Weather Bureau, Brisbane, Queensland, Australia.

Student Members

Rowlands, Michael, 'Sefton,' Thornholme Road, Sunderland, Co. Durham.
Shepperson, Alec, 32 Sutton Road, Kirkby-in-Ashfield, Notts.
Smith, Robert Michael, 87 Glen View Road, Habergham Eaves, Nr. Burnley, Lancs.

The following candidates were then declared to have been elected that day by the Council :

Fellows

Appleby, John S., 33 Bergholt Road, Colchester, Essex.
Chamuffin, Ronald Charles, 60 Worthington Road, Tolworth, Surrey.
Collis, Ronald, D.F.C., M.A., Decca Radar Ltd., 1-3 Brixton Road, London, S.W.9.
Dyer, Arthur, Ph.D., 17 Small Road, Noarabbin, Victoria, Australia.
Fox-Holmes, Basil, 'Maiscomb,' Hawthorn Road, Cherry Willingham, Lincoln.
Lang, David M., B.Sc., 9 Park Avenue, Chippenham. Wilts. (*transfer from Student Membership*).
Lawrence, George Richard Peter, B.Sc., Green Road, Whetstone, London, N.20.
Strong, Francis Harold Douglas, 71 Onslow Gardens, London, S.W.7.
Wood, Pilot Officer Evelyn M., Royal Air Force Technical College, Henlow, Beds.

Foreign Members

Gates, David Murray, Ph.D., M.S., B.S., c/o Scientific Liaison Officer, Office of Naval Research, Keysign House, 429 Oxford Street, London, W.1.
Hughes, James, 10,000 Woodhill Road, Bethesda 14, Maryland, U.S.A.
Lynott, Robert, 8821 N.E. Schuyler, Portland 20, Oregon, U.S.A.
Wurtele, Morton, Massachusetts Inst. of Technology, Cambridge, Mass., U.S.A.

Associate Member

Jackson, Rev. Graeme, B.A., 490 Havelock Road, Colombo 6, Ceylon (*transfer from Student Membership*).

Student Members

Allington, Kathleen R., Nutford House, Brown Street, London.
Bowers, Peter G., 13 Green Lane, Bexhill-on-Sea, Sussex.
Browning, Keith, 'Highlands,' Priors Hill, Wroughton, Swindon, Wilts.
Cox, Peter James, 85 Croft Road, Nuneaton, Warwicks.
Finch, Colin R., 80 Dickerage Road, Kingston-on-Thames, Surrey.
Frearson, Josephine, College Hall, Malet Street, London, W.C.1.
Fry, Jack Peter, 18 Pembury Grove, Bexhill-on-Sea, Sussex.
Griggs, Harry George, 11 Bedford Road, Earl Soham, Woodbridge, Suffolk.
Kemp, Alfred Kenneth, 47 Grosvenor Road, Coventry, Warwicks.
Martin, Geoffrey, Shoreditch Training College, Egham, Surrey.
Roberts, David, 63 Gregory Avenue, Green Lane, Coventry, Warwicks.
Sherman, Robin Lewis, 21 Holmbush Road, Putney, London, S.W.15.
Ventress, Margaret Anne, 18 Clarendon Place, Leeds 2, Yorks.

The following papers were read and discussed :

(1) 'Investigation into the accuracy of four types of rainfall recorder' by W. A. Lewis and L. H. Watkins (*Q.J.*, **81**, p. 449).

(2) 'The salinity of rainfall as a function of drop size,' by J. S. Turner (*Q.J.*, **81**, p. 418).

(3) 'The cloud-seeding trials in the Central Punjab, July-September 1954,' by E. M. Fournier D'Albe, A. M. A. Lateef, S. I. Rasool and I. H. Zaidi (*Q.J.*, **81**, p. 574). The paper was presented by Mr. F. H. Ludlam, and Mr. S. I. Rasool was present to answer questions.

Ordinary Meeting, 16 November 1955

The following candidates were declared to have been elected that day by the Council :

Fellows

Chamberlain, Arthur Cyril, M.A., Lynches Lodge, Whitchurch, Pangbourne, Reading.
Cooper, Leslie Hugh Norman, Ph.D., D.Sc., F.R.I.C., Marine Biological Laboratory, Citadel Hill, Plymouth, Devon.
Darbyshire, Jack, c/o National Institute of Oceanography, Wormley, Nr. Godalming, Surrey.
Garnett, Capt. Kenneth Trevor, c/o British European Airways, Ringway Airport, Cheshire.
Gentilli, Dr. Joseph, University of Western Australia, Nedlands, W.A., Australia.
Germani, F. O. Arthur, B.Sc., Halton House, R.A.F. Halton, Aylesbury, Bucks.
Hands, William Augustus, 'Wave Crest,' Marine Parade, Barmouth, Merioneth.
Hobbs, Peter, D.F.C., 103 Boyne Road, Lewisham, London, S.E.13.
McKeown, H. D. J., University College, Ibadan, Nigeria.
O'Carroll, Francis, 12 Dundela Avenue, Sandycove, Co. Dublin, Ireland.
Pennell, Walter Everitt, Waddington House, Waddington, Lincoln.
Rasool, Saiyed Ishtiaq, Laboratoire de Physique de l'Atmosphère, 1 Quai Branly, Paris, France.
Thambyahpillay, George, M.A., 14 Hope Street, Mill Road, Cambridge.

Foreign Members

Fisher, Edwin, M.S., 38-45 Northern Drive, Fair Lawn, N.J., U.S.A.
Gharbieh, Mohammad Arabi Abu, Chief, Meteorological Section, c/o Civil Aviation, Amman, Jordan.

Student Members

Adkins, Clement John, Christ's College, Cambridge.
Jones, Robert Graham, 32 Richard John Road, Milford Haven, Pembs.
Welburn, Thomas Hugh Norton, St. John's College, Cambridge.

The following papers were read and discussed :

(1) ' The effects of a large island upon the trade-wind air stream ' by Joanne S. Malkus (*Q.J.*, **81**, p. 538).

(2) ' The calculation of a sea-breeze circulation in terms of the differential heating across the coast-line ' by R. P. Pearce (*Q.J.*, **81**, p. 351).

(3) ' Theory of airflow over a heated land mass ' by R. C. Smith (*Q.J.*, **81**, p. 382).

Ordinary Meeting, 21 December 1955

The following candidates were declared to have been elected that day by the Council :

Fellows

Baxter, Rosalind Parkhurst, ' Chestnuts,' The Grove, Horley, Surrey.
Denison, Paul J., 910 46th Avenue, Lachine, Quebec, Canada (*transfer from Associate Membership*).
Fook, Foong Sze, 128a Caine Hill Road, Singapore 10, Federated Malay States.
Gates, Archibald Dunham, 315 Bloor Street West, Toronto, Ontario, Canada.
Hayward, Beryl, 22 Westbrook Road, London, S.E.3 (*transfer from Student Membership*).
Hwang, Tiaw Sooi, 13 Joo Seng Road, Singapore 13, Federated Malay States.
Kaffetzakis, John George, 315 Bloor Street West, Toronto, Ontario, Canada.
Kelly, George Milton, Moorcroft, Green Lane, Crowborough, Sussex.
Mather, Richard, c/o Malayan Meteorological Service, P.O. Box 715, Singapore, Federated Malay States.
McDonald, Alistair, c/o Malayan Meteorological Service, P.O. Box 715, Singapore, Federated Malay States.
Mettrick, Harold, 315 Bloor Street West, Toronto, Ontario, Canada.
Osborne, David, 50 Monk Road, Bishopston, Bristol 7, Somerset (*transfer from Student Membership*).
Pill, John, ' Devonia,' 32 Kingsholm Road, Southmead, Westbury-on-Trym, Bristol, Somerset (*transfer from Student Membership*).
Rajendram, Kandiah, 15 Cassia Drive, Raffles Park, Singapore, Federated Malay States.
Rutenberg, Anatol, 315 Bloor Street West, Toronto, Ontario, Canada.
Slatyer, Ralph Owen, M.Sc., C.S.I.R.O., Canberra, A.C.T., Australia.
Smith, Ralph Charles, M.Sc., Ph.D., 54 Wallace Crescent, Chelmsford, Essex (*transfer from Student Membership*).
Spackman, Ralph Sadler, 15 Byron Avenue, Couldson, Surrey.
Trecarten, Lewellyn Allwyn, 315 Bloor Street West, Toronto, Ontario, Canada.
Van Ark, Willem Bernard, 356 Horner Avenue, Toronto, Ontario, Canada.
Vickers, Donat Osmond, M.Sc., c/o The British Caribbean Met. Service, P.O. Box 461, Port of Spain, Trinidad, B.W.I.
Whitfield, Charles, B.A., 1 The Hermitage, Summer Hill, Harbledown, Canterbury, Kent.

Foreign Members

Hardy, William Eric, Dept. of Meteorology, Oklahoma Agric. & Mech. College, Stillwater, Oklahoma, U.S.A.
Macoosh, Gabriel S., 598 Kiryat Shalom, Tel Aviv, Israel.

Student Members

Hamilton, Paul Marshall, 24 Middle Drive, Rednal, Birmingham, Warwicks.
Jones, John Conrad, 67 Langley Way, Watford, Herts.
Smith, Brian Maurice, 1 Waterloo Place, Cranbrook, Kent.

The following papers were read and discussed :

(1) ' Free and forced convection in the atmosphere near the ground ' by C. H. B. Priestley (*Q.J.*, **81**, p. 139). This paper was presented by Sir Graham Sutton, F.R.S.

(2) 'An operational frontal contour-analysis model,' by R. Anderson, B. W. Boville and D. E. McClellan (*Q.J.*, **81**, p. 588).

Popular Lectures

11 October 1955. Mr. C. J. Boyden being in the Chair, Mr. F. H. Ludlam repeated his Popular Lecture on ' Clouds ' at the Luton Girls' High School. The Lecture was open to sixth-form pupils, of whom over 200 were present by arrangement with the Education Officers of the Borough of Luton and the Bedfordshire County Council.

25 November 1955. The President, Dr. R. C. Sutcliffe being in the Chair, Mr. P. J. Meade gave the first of the Society's series of Popular Lectures for 1955-56, his subject being ' Fog and atmospheric pollution.' The Lecture was open to sixth-form pupils and was held in the Conference Hall of the London County Council by kind invitation of the Chief Education Officer of the L.C.C. By arrangement with the Education Officers of Kent and Surrey, pupils from these areas were also present.

8 December 1955. Dr. R. S. Scorer being in the Chair, Mr. P. J. Meade repeated his Popular Lecture, which was again held at the Conference Hall of the London County Council. By arrangement with the Education Officers of Buckinghamshire, Essex and Hertfordshire, pupils from these areas were also present, together with a number of pupils from independent schools. The total attendance at the two lectures at the Conference Hall was about 700.

Centenary Celebrations at Scottish Centre

The Centenary of the founding on 11 July 1855 of the Scottish Meteorological Society was marked by an exhibition entitled ' Scotland's Weather 1855-1955 ' held in the Royal Scottish Museum, Edinburgh, from 6 June to 2 July, to which there were over 14,000 visitors. A full account of the opening ceremony and the exhibits appeared in the November 1955 issue of *Weather*.

Lecture and Dinner on 28 October. The President, Dr. R. C. Sutcliffe, being in the Chair, a meeting of the Scottish Centre was addressed by Sir Ernest Wedderburn, Vice-President for Scotland and Chairman of the Scottish Centre, his subject being ' The Scottish Meteorological Society.' After the Lecture, a Dinner was held in the Rooms of the Royal Society of Edinburgh, with an attendance of about 30 Fellows and guests. The President and some of the Officers of the Society were present and Sir Ernest Wedderburn was in the Chair.

Boundary-layer theory. By H. Schlichting, trans. by J. Kestin. London (Pergamon Press), 1955. Pp. x, 535; 295 Figs., 35 Tables. 105s.

In 1904, Ludwig Prandtl read a paper on the motion of fluids with very small friction before the Mathematical Congress at Heidelberg, and in doing so bridged, for the first time, the gap between classical hydrodynamics and the technology of hydraulics. In this paper he made explicit the concept of the boundary layer, namely, that when a fluid of low viscosity (such as air) moves over a solid body, friction plays an essential part only in a very thin layer of air immediately adjacent to the body, and that elsewhere the fluid may be treated as inviscid.

There is little doubt that this idea had occurred to others as well – it is implicit, for example, in much of Lanchester's semi-intuitive aerodynamics – but Prandtl was the first to state the concept explicitly and to show that this physical hypothesis rendered the Navier-Stokes equations tractable for a large class of problems. Before this, the equations of hydrodynamics had been employed either for academic problems in inviscid fluids or in certain limiting cases of extremely slow motion in a viscous fluid, such as Stokes's famous solution for the ' creeping ' motion of a sphere. The lead given by Prandtl had a tremendous effect and from 1904 onwards the boundary-layer theory became the subject of intensive research. It is no exaggeration to say that without it aerodynamics could hardly have been developed as a respectable science, particularly in relation to drag.

The present volume is the work of one of the best known investigators in this field, and is based upon a series of lectures published in Germany in 1942. After the war, Dr. Schlichting revised the original text and enlarged it to include later advances, as well as much material classified as ' secret ' by the German authorities. The present volume is a translation from the German text of 1951.

The style of the work is very much that of the familiar German ' Handbuch ' and the author makes it clear from the beginning that the book is intended primarily for ' engineers working in industry and in research establishments and for students of mechanical engineering and machine design.' The text falls into four main parts. The two introductory chapters are descriptive, an exposition of the essential ideas of boundary-layer theory, with the minimum of mathematics. Chapters III to XV, occupying 276 pages, cover the development of the theory of motion in a laminar boundary layer, beginning with the derivation (in full) of the Navier-Stokes equations, their general properties and the few exact solutions (e.g., those for parallel flow) that are known. Chapter VI discusses very slow motion and problems of lubrication.

The first six chapters thus contain material to be found in any good text on hydrodynamics, but in much greater detail than usual. The techniques of boundary-layer theory are developed in Chapters VII to XV. The titles of these six chapters are sufficient indication of their content. The headings are: equations for two-dimensional flow; general properties of the equations; exact solutions of the steady-state equations for two-dimensional flow; axially symmetrical and three-dimensional boundary layers (e.g. jets); non-steady boundary layers; approximate (numerical) methods of solution. The text is very complete and detailed. Chapter XIII describes techniques of ' boundary layer control ' (e.g. by suction, to prevent early transition or separation). Chapter XIV, one of the most interesting in the book, is a well-written account of the thermal boundary-layer in laminar flow. Finally, Chapter XV brings the account of laminar boundary-layers to a close with a discussion of the influence of compressibility, and includes some very fine Schlieren photographs of high-speed flow.

The development of boundary-layer theory for laminar flow is now regarded as practically complete, and this section of the book may well become the authoritative text. It is doubtful if any contribution of significance has been omitted.

The third part of the book deals with the notoriously difficult problem of the transition from laminar to turbulent flow. Here the text begins to take on the character of an original work, as distinct from an integrated account of the work of many hands, for the author was a pioneer worker in this field. The early Tollmien-Schlichting mathematical analysis predicted that in laminar flow, small perturbations of a certain wavelength and amplitude would be amplified if the Reynolds number exceeded a certain low value (about 500). This result was opposed to other theories which required transition to originate from finite disturbances in the flow outside the boundary layer. For many years all the experimental evidence supported the 'finite disturbance ' hypothesis, but in 1940, Schubauer and Skramstad, working with a wind tunnel of very low turbulence at the Bureau of Standards, U.S.A., found the Tollmien-Schlichting waves in the flow near a flat smooth plate, and the mathematical theory of transition induced by small oscillations was triumphantly verified.

The final section of the book deals with turbulent boundary layers. The treatment is restricted to the semi-empirical methods developed for practical problems relating to the frictional drag of smooth and rough surfaces, and with aerodynamical applications. The mathematician will find the discussion inadequate – the formation of mean values, for example, is dismissed in a few lines of dogmatic statements (with one unfortunate misprint that makes nonsense of a fundamental postulate) – and the more recent developments, such as the statistical theories, are hardly mentioned. As might be expected, the mixing-length hypothesis is given pride of place. Since the book is written primarily for engineers, the restriction to empirical methods can be justified, because so far none of the deeper theories has yielded much in the way of working formulae, but one is left with the feeling that a book of this size might have devoted more space to a discussion of the more rigorous analysis that is now being developed.

How far can this account help the meteorologist ? Boundary layers are no novelty to the meteorologist, but as yet there has been little opportunity to use the more precise parts of the theory in problems connected with air movements over the ground. The book is a mine of information on the drag of flat plates, bodies of revolution, pipes, rotating discs and aerofoils, and anyone who wishes to probe these matters could not do better than to refer to these pages. For the most part, however, the meteorologist will find the essential facts about surface friction, wind profiles and mixing-length theories adequately summarized in his own professional texts, and he will not get from this book any real enlightenment on certain matters of prime importance in his science. Thus there is no discussion on the diffusion of matter, and only a very brief account of the suppression of turbulence by a density gradient, the phenomenon being dismissed as ' of some importance in certain meteorological processes.' The Richardson number is mentioned and defined, but ascribed to Prandtl, and, in general, problems connected with the earth's boundary layer are omitted. One cannot complain about this since it is made abundantly clear throughout that the book is primarily intended for engineers, particularly those working in aeronautics.

The general layout of the book is good and the reproduction of the photographs and line diagrams calls for high praise. Considerable typographical skill has been shown in the setting of the many complicated mathematical expressions, but the press work is hardly up to the standard to be expected in so costly a book and there are many blemishes in the printing of both text and formulae. None of these, however, is sufficiently serious to confuse the careful reader.

O.G.S.

The Polar aurora. By Carl Störmer. Oxford University Press, 1955. Pp. xvii, 403; 216 Figs., 27 Tables. 55s.

This book by the world's leading authority on the aurora is one of the series of International Monographs on Radio. The author's work has been as distinguished in the field of observation as in that of theory – the result, as he writes in the preface, of his being both a pure mathematician and a keen amateur photographer – and these two aspects figure with equal prominence in this book.

Our present knowledge of auroral characteristics is largely based on the unrivalled series of observations organized by the author at Norwegian stations over a period of more than 40 years. Here the techniques which have been developed are given in detail and the results obtained are analysed. Visual and photographic methods of measurement of the auroral spectrum are described, and separate short chapters are devoted to the attempts which have been made to obtain radio echoes from solar particles outside the earth's atmosphere and from the aurora itself. In this latter connection workers at Jodrell Bank are convinced that they have found true auroral echoes but have not yet won unanimous support for their view. The author concludes the first part of his book with discussions of the mechanism of ejection of particle streams from the sun and of their absorption in the earth's atmosphere.

In the second part of the book Professor Störmer looks for a theory which will satisfactorily account for at least the main features of the aurora which are revealed by observation. His own classical studies of the trajectories of electric corpuscles in a magnetic field naturally occupy a prominent place in this discussion. A full description of the Störmer theory is given, including details of its applicability to aurora in its many varied forms. The main objection that has been raised to this theory is that mutual electrostatic repulsion of the particles comprising a stream of solar particles of one sign would cause the stream to disperse before arrival at the earth : a similar objection applies to the equatorial ring current which plays an essential part in the theory. The criticism is difficult to refute, but Störmer is not convinced of its validity : certainly one has the feeling that his theory produces so many results in close accord with actual observation and with experiment that it must be soundly based.

Most subsequent workers in auroral theory, or the closely allied theory of magnetic storms, have met the major criticisms made against Störmer's work. The author reproduces verbatim accounts of the two most notable theories which have been carried as far as the auroral process – those of Martyn and Alfvén – but makes no comment on them other than that Martyn's theory does not account for some of the characteristic auroral forms, and that Alfvén's theory rests on the existence of a general magnetic field of the sun, which is yet unconfirmed. This relative lack of comment is to be found in other parts of the book and is a feature which is likely to disappoint some readers.

Even those unfamiliar with the subject will find this a very readable book. Frequent use is made, especially in the first part, of extracts from original accounts and this has the effect of making the book particularly alive. The work does not cover every aspect of the aurora but this could hardly have been achieved within one volume and nothing of major importance is omitted. In no previous work on the subject has a detailed description of the basic facts and techniques, combined with a clear statement of the problems confronting the theorist, been so well achieved as in this book.

D.H.M.

Klima und Bioklima von Wien. I, Teil – Ergebnisse der langjährigen Messreihen an der Zentralanstalt für Meteorologie und Geodynamik in Wien, Hohe Warte. By F. Steinhauser, O. Eckel and F. Sauberer. Austrian Met. Soc., Vienna, 1955. Pp. 120; 2 Figs., 122 Tables. Np.

This small volume, which presents the climate of Vienna in extensive tables of figures, is the first part of a work addressed to the needs of town planning and industry. The work is commissioned by the city

council and derives assistance from a bioclimatology working group of the Austrian research society dealing with problems of urban life and development.

There is much that is new in the multifarious presentation and may serve as a model to those dealing with similar problems elsewhere. The only shortcoming is that surprisingly few of the facts are given in diagram form; and the two sheets of diagrams are loose and liable to get lost.

The climatic records of the official observing station at the headquarters of the Austrian meteorological service are presented month by month from 1851 to the end of 1954, corrections having been applied to cover a change of site in 1872. An older record going back to 1775 in the neighbourhood of the university supplied details of earlier extremes of temperature : the extremes, both high and low, of monthly mean temperature mostly come from the earlier period – a fact attributed to the sheltered position of the Zentralanstalt since 1872, which moderates the extremes produced by radiation and wind.

The tables are a monument of diversity designed to cater for numerous needs, including, besides all the obvious elements and parameters, such items as height of the sun each hour of the day, frequency on each day of the year of rainfall exceeding 0·1, 1·0 and 5 mm, average hourly values of relative humidity on clear days and on all days, hourly frequencies of calms, light winds and strong winds from the prevalent directions in different months of the year.

Economy of descriptive text means incidentally that the facts are open to anyone with a German dictionary. Indeed, the bewildering wealth of figures may be less intimidating to the foreign meteorologist than to the local man who is only wondering about a site for factory or dwelling. Later parts of the work are planned to deal with microclimatic patterns within the area of the city and environs.

The work should find its way into all the leading meteorological, geographical and town-planning reference libraries in Europe.

H.H.L.

Sun, sea and sky. By Irving P. Krick and Roscoe Fleming. London (Victor Gollancz), 1955. Pp. 248; 11 illustrations and 9 weather maps. 16s.

As the title suggests, this book is probably intended more for the reader who gets his science from the popular magazine than for the serious meteorologist : ' Rivers in the sky ' is the heading of the first main section, with chapters like ' Report from space,' ' Smiling Magician,' ' Life's umbrella ' and ' Greenhouse with a blue roof.' In spite of the journalistic flavour and a striving after freshness of presentation, the book succeeds in describing a wide range of everyday phenomena of weather and climate in an easy, convincing and memorable manner.

As the senior author, according to the book's jacket, was until 1948 head of the department of meteorology at the California Institute of Technology, we expect the contents to be meteorologically sound and unprejudiced : as Dr. Krick has also been head of the Water Resources Corporation of the United States since 1950 we can expect an illuminating description of the way meteorology is applied in the interests of U.S. industry and agriculture. We certainly get the latter. Quoting the recommendations of the George Committee we learn that the relation between the national weather service and the free-lance meteorologist should be like that between a public health service and the physician. The health service is concerned with the general ways of protecting and improving public health; the special services to industry should be the province of the private practitioner. And according to the description of a day in the life of an industrial meteorologist (' Predictions, public and private ') it seems to work like that in the United States.

Opinions will differ about the meteorological soundness of some of the notions. We are told in the section on ' Tools of the weatherman's trade ' that the ' analog ' (or ' synthetic weather-memory ') system of forecasting provides the most effective guide to the weather of tomorrow, next week, next month, next year; later, in ' Weather and war ' we learn that it was this ' analog ' method which alone chose 6 June 1944 as providing the only possible weather for the allied invasion of Europe. That, to speak mildly, is hardly likely to be an acceptable statement of what took place : it is not even worthy of Dr. Krick as a professional meteorologist, and as one who contributed honourably to the D-day forecast. During those early days of June 1944 the number, intensity, speed, rate of development and complexity of the weather systems which moved towards the British Isles could not be matched, even as regards their superficial features, on any summer chart in the previous 40 years. Even short-range forecasting methods were taxed to the limit. Indeed, it was almost a matter of pure chance that a brief interlude, sufficiently good to allow the vast operation to get under way, supervened so quickly after the invasion had been postponed.

J.M.S.

Deterioration of materials. Edited by Glenn A. Greathouse and Carl J. Wessel. New York (Reinhold), 1954. Pp. xvii, 835; 254 Figs., 57 Tables, 1 Plate. 96s.

' Lay not up for yourselves treasures upon earth, where moth and rust doth corrupt . . . ' St. Matthew, 6.19.

Although he may not have heeded the Apostle's advice, Man has endeavoured throughout the years to overcome the costly damage to his property which can be caused not only by moth and rust but by many other biological and chemical agents. In particular, he has had to contend with the effects of meteorological conditions, and in this respect, he has made things worse in many places by converting fog into ' smog.'

Many books have been written on the various aspects of deterioration of materials, but this well-illustrated book represents a very fine effort to cover the whole field. Originating from wartime requirements, a programme was initiated in the United States under the Office of Scientific Research and Development with the object of overcoming the deterioration of equipment and materials of all kinds. This led to the development in this particular field of a school of scientific workers and the results of their work and accumulated knowledge during and after World War II are gathered together in this volume. It represents an excellent co-operative effort between the Services Technical Committee of the U.S. Department of Defence and the Prevention of Deterioration Centre, Division of Chemistry and Chemical Technology, U.S. National Academy of Sciences – National Research Council.

In view of present-day economic problems – deterioration of materials costs millions of pounds each year – the publication of this book is both valuable and timely. It consists of four parts. The first part covers the scientific aspects of deterioration and deals with climatic, chemical, physical and biological factors. It is appropriate that the opening chapter should relate to climate; it outlines the world distribution of climatic types based on Trewartha's modification of the Köppen system – the authors admit that, for workers on problems of deterioration, this system may present difficulties – and discusses, with many examples, the effects of various climates on materials. The following chapter on chemical and physical agents is also of interest to the meteorologist in that it relates to the effects of radiation, oxygen and ozone, impurities in the atmosphere, moisture, heat, cold and various weather phenomena. In the second and third parts the prevention of deterioration is discussed for specific materials, i.e., metals, wood and wood products, paper, textiles and cordage, leather, plastics and rubber, paints, varnishes, enamels and lacquers, and also for assembled units, such as electrical and electronic equipment, optical instruments and photographic equipment. The last part covers special aspects of preservation such as dehumidification, recommends packaging and storage techniques and gives methods for assessing the toxicity of preservatives.

As is modestly stated in the editors' preface, it is not pretended that the book provides solutions of all deterioration problems; only a broad outline of the whole field of prevention is given. Nevertheless, the book contains a wealth of information with extensive references to the relevant literature. It is admirably produced and will be of interest to all those who are concerned with the design, manufacture, transport, storage and maintenance of equipment in all its various forms. The meteorological consultant should certainly have a copy among his books of reference.

R.G.V.

Das Eiszeitalter. Grundlinien einer Geologie des Quartärs. By Paul Woldstedt. Bd. 1. Die allgemeinen Erscheinungen des Eiszeitalters. Stuttgart (F. Enke), 2nd. Ed., 1954. Pp. vii, 374; 136 Figs. 41 DM.

This book is a mine not only of facts about present and Quaternary ice sheets and glaciers but also of the many questions which are still subjects for controversy. After a historical introduction and a summary of the approach of the Ice Age – following a gradual fall of 10°C in central Europe since the Eocene, temperature slumped suddenly at the beginning of the Pleistocene – the next chapters describe existing ice-sheets and glaciers, periglacial formations, permafrost and glacial erosion. The opinions are expressed that the Greenland ice-sheet is not the cause, but the consequence, of the low temperature – a rather meaningless dictum – and that the Antarctic ice-cap is larger than is warranted by the present climate.

Chapters 6 to 9 give accounts of erratics, melt-water deposits and varves, periglacial floods and eolian deposits. Gravel terraces are attributed to the stage of advancing glaciers and new methods of inferring the climate during their deposition are described. Loess is regarded as the result of double sifting, first by water and then by wind, and deposition on a grass steppe at the peak of glaciation. A short account of interglacial and interstadial formations and climates is followed by a correlation of glacial periods in different regions and the remarkable series of sea levels in the Mediterranean. After chapters on fauna and flora, human cultures, earth movements and river terraces, we come in Chap. 16 to the climate of the ice age, which has been the subject of much recent reconstruction. The indications are rather contradictory, for while the European ice sheet was surrounded by a wide belt of tundra in which the mean temperature was below — 2°C, the water flora is often temperate, and the depression of the snow line is less than would be expected from the fall of temperature. This is accounted for by a decrease of precipitation, especially in the dry subtropical and polar regions. Even the type of circulation has been inferred : a shift of the upper trough eastwards and a stronger and more meridional circulation. The climatology of the Ice Age has become dynamic.

With such a wealth of material, one would expect the causes of the Ice Age to present no difficulty, but it is far otherwise; new theories spring up faster than old ones are discarded. The last chapter, of 21 pages, divides them into five groups; terrestrial, astronomical, the Köppen – Milankovitch hypothesis, pole movements and solar or cosmic causes. Since the Ice Age was world-wide all *local* causes such as a shift of the Gulf Stream, or even mountain building, are excluded, though Wagner's hypothesis of a decrease of radioactive heat through mountain-building is regarded as plausible. ' Astronomical ' theories, which reached their culmination with Milankovitch, are treated at considerable length and numerous inconsistencies are pointed out. It may be added that since this book was written, a recalculation of the astronomical factors

by Brouwer and van Woerkom has made them even less credible as factors of climatic change. Pole movements are not regarded as sufficient (recent research is bringing them into the picture again !). The numerous theories invoking changes of solar radiation include recent hypotheses of changes in the ultraviolet and corpuscular radiation; again the verdict is ' not proven.' In a brief summing-up the author concludes that the problem is not yet soluble; it must take account not only of the Quaternary Ice Age but of the whole long series of geological climates.

C.E.P.B.

Ergebnisse und Probleme der Sonnenforschung. By M. Waldmeier. Leipzig (Geest & Portig), 2nd. Ed., 1955. Pp. xi, 389; 146 Figs., 68 Tables. 38 DM.

Solar physicists and their students will welcome the appearance of a new and revised edition of *Results and problems of solar research.* It has long been known to them as a valuable source book; but the circumstances of its publication – in a limited German edition, printed during the early days of the war (1941) – made it virtually unobtainable in England and America. The only copy of the first edition known to the reviewer was a microfilm version in the Library of the Royal Astronomical Society.

Prof. Waldmeier holds an honoured position in solar studies; this is partly due to his own massive achievements and partly to his balanced and cautious interpretation of the observational results.

The book itself contains twelve chapters, covering in a comprehensive manner nearly all branches of solar physics. In the first four chapters the author introduces us to such classical problems as the intensity distribution within the continuous spectrum, the solar constant, and the derivation of the temperature of the photosphere. He goes on to summarize the theoretical work on the internal constitution of the sun and the nature of its energy sources. The Fraunhofer spectrum is considered very fully in Chapter 5, together with a discussion of the theory of line profiles, curves of growth, and the problems concerned with the chemical analysis of the reversing layer.

The four chapters which follow relate to the sun's atmosphere and its special types of activity, as represented by sunspots, faculae, chromospheric flares and prominences.

Chapter 10, on the corona, is perhaps the most valuable section of the book, since it distils within the compass of 40 pages the essence of many original papers relating to the sun's outer atmosphere which have appeared during the past two decades.

The two final chapters found no place in the first edition; they relate to new branches of the subject, namely solar radio emission and solar-terrestrial relations. The last chapter is, perhaps, too short to deal adequately with all that is known of the relationship between solar activity and the ionosphere; one could have wished, in particular, for a fuller treatment of solar flares and their terrestrial effects.

The book is excellently illustrated with plates and diagrams of the highest quality; these, together with the copious references to original papers, make it a *vade-mecum* for the research worker.

M. A. E.

Amateur astronomer's handbook. By J. B. Sidgwick. London (Faber and Faber), 1955. Pp. 580; 193 Figs., Tables. 63s.

It can be fairly safely said that this book – together with its companion volume *Observational astronomy for amateurs* – is the first attempt to embrace all aspects of amateur astronomy within two volumes since the publication of Hutchinson's *Splendour of the heavens* some thirty years ago. Mr. Sidgwick has therefore rendered a considerable and long-needed service to amateur astronomers by the completion of his monumental task. The general impression of the book is good. The chapters and sections are well and logically set out, and adequately indexed and numbered. The figures, apart from the graphs, are generally clear and well-drawn. The drawing of the graphs, however, is the one glaring blot in the layout. In several of them points on an obviously smooth curve are joined by straight lines, and where curves are drawn they are often clearly unsatisfactory.

The first half of the book is devoted to an exhaustive discussion of all aspects of the astronomer's main tool, his telescope. Particularly welcome is the chapter on oculars, which – at least in the reviewer's experience – is about the only adequate and up-to-date discussion of this vital component of the telescope in existence. Then follow chapters on micrometers, spectroscopes, photography, photometry, and a particularly interesting chapter on vision and the performance of the eye under all conditions. In the chapter on atmosphere and seeing, the author seems to venture on unfamiliar ground, for there are several inaccuracies and obscurities. In particular, the section on atmospheric humidity is quite inadequate.

However it would be invidious to dwell unduly upon a few defects in an otherwise sound work, which concludes with a chapter on time and clocks, miscellaneous short articles, a comprehensive and well arranged bibliography of nearly 600 books and references, and a table of contents of *Observational astronomy for amateurs.*

W.T.R.

Statistical methods for research workers. By Sir Ronald A. Fisher. Edinburgh (Oliver and Boyd), 12th Ed., 1954. Pp. xv, 356; 12 Figs., 74 Tables. 16s.

This monograph, when originally published in 1925, was a pioneering work on the practical applications of the theory of small samples. By the 1930s, however, it had become a standard statistical text book, and since then the underlying ideas have spread to every field of knowledge in which statistical methods can be applied. The treatment starts from fundamentals, including the binomial, the normal, and the Poisson distribution, and discusses tests of goodness of fit based on the x^2 distribution and various tests of significance based on the t- and z-distributions. The correlation coefficient and the analysis of variance are discussed and the exposition finishes with a chapter on the principles of statistical estimation.

The author has probably done more than any living person to raise the status of statistics as a branch of applied mathematics, and a book by him which has fulfilled its purpose for a generation without major addition or amendment can lack little in the way of soundness or comprehensiveness. Nevertheless, it is a difficult book to read or master, and most workers wishing to apply statistical techniques in some well understood and circumscribed field would be well advised to start on something easier. However, the meteorologist who wishes to apply exact statistical methods to any of the more interesting branches of the subject will not find himself in this happy position. If he wishes to progress beyond simple description when studying synoptic meteorology, climatology or the large-scale turbulent processes comprising the planetary circulation he soon finds himself facing a statistical Everest, in which the approach to formidable theoretical problems is barred by the near impossibility of processing enough data to show clearly the essential character of the entities with which he has to deal. The anthropologist intending to carry out a statistical investigation of men has no difficulty in obtaining an adequate sample of men without including any women or gorillas, but the meteorologist examining Atlantic depressions or cold winters over Western Europe has no reliable means of ensuring that his cases are drawn from the same population. If, as he is bound to, he applies the most rigorous criteria to ensure that they are, then his samples will be small and often not mutually independent.

The worker who wishes to distinguish fact from fancy in this field has two special requirements. The first is access to computing facilities on a massive scale, to enable him to convert and condense a myriad of raw data into a smaller number of significant measurements which the mind can compass. The second requirement is the very clear understanding of statistical principles needed to draw true inferences from small samples of intractable material, and to obtain this understanding he can look to no better source than the book now under review.

J.M.C.

Man in a cold environment. By A. C. Burton and O. E. Edholm. London (Arnold), 1955. Pp. xiv, 273. 75 Figs., Tables and Illustrations. 30s.

Newton's cooling law tells us that we must lose heat whenever the temperature of our surroundings is less than 37°C, and as it is usually below that in these latitudes we wear clothes. What clothes we wear depends on how far our surroundings are below body temperature, what we are doing, what the wind speed is and, perhaps, how damp the air is. All this is everyday experience : but, as with so many everyday things, we are apt to find that a closer look into the physics of the business leads us into other fascinating subjects, seemingly far removed from the starting point. That is what this book does for us. For although the authors devote the second half to a rigorous and well-authenticated account of recent work on the reactions of the body and bodily processes to extreme cold – and so justify their sub-title, ' Physiological and pathological effects of exposure to low temperature ' – in the earlier chapters which describe the mechanisms of heat transfer from the human body to the surroundings, they do not shrink from following side tracks of much wider interest. For instance a discussion of the remarkably narrow limits of body temperature throughout the species of warm-blooded animals – from 36°C for elephants, through 37°C for humans to 41°C for birds – leads to an account of the mechanism by which the body temperature is regulated and to speculation about the reason for settling on 36°-41°C and hence about the regions of the earth where the warm bloods first developed that mechanism. Again, it is an easy step from the 'dead-air' spaces that provide the insulation in the fur of animals and human clothing to an assessment of the best size of cloth mesh and material for protection; it is not much further from this to the important difference between the conductivity of still air in small packets and the very much smaller value given in tables of physical constants. From another section the amateur plumber can learn why the lagging of small-diameter pipes can easily be overdone; after a particular curvature (not much below that of the pipe itself) has been exceeded an increase of the radiating surface may counter-balance the insulating properties even of the most suitable material.

Throughout the book there is much to interest the meteorologist professionally as well as privately, provided he does not let himself be put off by words like poikilotherm and homeotherm or a unit of insulation like the clo. He will be impressed even more than he probably is already with the necessity of thinking in terms of tables of contingency rather than of isolated frequencies, especially if he has to advise on conditions of living in other climates. It leads only to unnecessary encumbrance – often at the expense of more valuable equipment – if people are advised that they will encounter temperatures down to – 45°C if at the same time they are not also told that these low temperatures will occur only on a few isolated occasions when the wind is light.

J.M.S.

Radio astronomy. By J. L. Pawsey and R. N. Bracewell. Oxford University Press, 1955. Pp. x, 361; 150 Figs., 30 Tables, 23 Plates. 55s.

During the past decade rapid progress has been made in radio astronomy both in the experimental techniques in use and in the results achieved. Much of this material has never been discussed in a comprehensive survey. As the importance of radio astronomy in relation to other branches of astronomical research has become increasingly evident, the need for such a survey has been felt by both astronomers and physicists. The present volume satisfies this need in a manner marked at once for its detail and its clarity.

The authors are associated with the Radiophysics Laboratory, Sydney, a station where many lines of research in radio astronomy have been pursued, and they are therefore particularly well qualified to discuss the merits of the observational techniques which are treated in the first section of this book. The main body of the book is devoted to the observations of the solar and cosmic radio emission. The chapters on these topics are accompanied by accounts of the information relevant to radio astronomy which is provided by optical measurements. This feature of the work is particularly welcome since the interpretation of the radio observations depends so greatly on our knowledge of the Universe derived from observations at optical wavelengths. Other chapters are concerned with the theory of radio waves in ionized gases, the thermal radio emission from the moon, and a brief survey of meteor physics.

Whilst it has been possible to achieve a comparatively up-to-date survey of experimental methods, the authors have attempted to provide a complete account of the observational results only up to the middle of 1952. Branches of radio astronomy such as the observations of the 1420 mc/s spectral line from neutral hydrogen, which have subsequently developed into extremely important and extensive studies, are therefore treated quite shortly. This limitation, inevitable in a rapidly changing subject, has meant that this book is chiefly valuable as the only comprehensive account of the fundamentals of the subject. The presentation is aided throughout by numerous diagrams and the result is a work which one hopes will set the standard for future monographs on radio astronomy.

J.E.B.

Obituary Notices

Mr. E. W. BLISS, M.A., M.Sc., F.Inst.P.

Ernest William Bliss, a Life Fellow of the Society, elected on 18 March 1914, died suddenly at his Worcestershire home on 4 August 1955 in his 67th year. After graduating at the University of Birmingham he won an open scholarship to Christ's College, Cambridge, and there showed high abilities as a mathematician. He was awarded first-class honours in Part I of the Mathematical Tripos in 1909 (a vintage year for future meteorologists, as it turned out, with the names of David Brunt and Sydney Chapman appearing in the same class-list) and was one of the 25 Wranglers when he took Part II of the Tripos in 1911. Later he did good work as a scientist in the Survey Department of Egypt. Returning to England in 1924, he embarked on research at the Imperial College of Science and Technology, South Kensington, and during subsequent years collaborated in meteorological investigations with Sir Napier Shaw and Sir Gilbert Walker, besides pursuing various lines of inquiry on his own account. Among his special interests were world weather relationships and their possible use in seasonal forecasting – a subject on which the series of papers, written either by himself or jointly with Sir Gilbert Walker, for this Society's series of *Memoirs* are known internationally as classical studies of their kind. Perhaps Bliss's own most noteworthy finding, deduced from evaluation of more than 300 correlation coefficients (1926), was that certain prior conditions in the earth's southern hemisphere apparently play a part comparable with that of the North Atlantic oscillation in controlling the winter weather of the British Isles.

E.L.H.

Group Captain W. N. CUMMING, O.B.E., D.F.C.

We have to record the death, on 29 June 1955, at the age of 56, of William Neville Cumming, who had held Fellowship in the Society since 17 April 1935. A native of Edinburgh and educated at George Watson's College in that city, he was attracted by aviation while still a boy; from the later stages of the 1914-18 war until 1926 he served with the R.N.A.S. and the R.A.F. He then went to Canada, where he undertook much pioneer civil flying during the next four years. In 1931 he joined Imperial Airways, soon winning distinction as a skilled pilot of long and varied experience. He flew principally on the African and Mediterranean routes, and when carrying out the initial tests of the Short *Caledonia* aircraft he made a non-stop flight from Southampton to Alexandria (4 March 1937). On the outbreak of the 1939-45 war he returned to the R.A.F., serving first in Coastal Command and subsequently as staff officer in charge of anti-U-boat defences in the Indian Ocean. In 1949 Group Captain Cumming became Master of the Guild of Air Pilots and Navigators of the British Empire. At one time he was chairman of the British Air Charter Association.

E.L.H.

Col. A. M. GIBB, T.D.

Colonel Alistair Monteith Gibb, a Fellow of the Society since 1946, died on 29 July 1955 from the effects of an accident sustained while playing polo. Born in Westminster on 2 April 1901, he was educated at Eton and Pembroke College, Cambridge, and after completing his pupilage under the late Sir Basil Mott joined his father's firm of consulting civil engineers (Sir Alexander Gibb and Partners), gaining his early professional experience in Newfoundland, India and Singapore. Later his work took him repeatedly to Kenya. His knowledge, there acquired, he subsequently turned to advantage as a member of the executive council of the Joint East and Central Africa Board, as deputy chairman of the Conservative Commonwealth Council, and as chairman of the East and Central Africa Group of that council. During the 1939-45 war Gibb served in the Middle East and Italy with the Royal Wiltshire Yeomanry, holding the rank of lieutenant-colonel from 1942 to 1944. At the battle of El Alamein he took over command of the regiment when his senior officer had been wounded. In 1951 he was made honorary colonel. He re-joined his father's firm in 1944 and became senior partner in 1948. His interest in meteorology arose from practical experience of farming in Kenya and from the need for accurate and detailed climatological knowledge in the designing of civil engineering works to be established abroad. For the bulk of the information given in this notice we are indebted to Sir Alexander Gibb, who has held Fellowship in the Society since 1912.

E.L.H.

Mr. F. E. LEE

A Fellow of 35 years' standing was lost to the Society by the death on 28 September 1955 of Mr. Francis Eric Lee, at the age of 65. Throughout a life of farming at Tetchill, Ellesmere, Shropshire, Mr. Lee had taken a keen interest in meteorology. His studies enabled him to achieve no small reputation locally as a reliable weather forecaster.

E.L.H.

Captain TUDOR WILLIAMS, R.N.R.

Information has recently reached the Society that Captain Tudor Williams, a Fellow since 17 January 1951, died suddenly at his home near Plymouth on 30 December 1953, in his 48th year. At the age of 15 Williams went to sea as an apprentice with the Cardiff Shipping Company (Sir W. Reardon Smith & Sons). During the 1939-45 war he received a lieutenant's commission and served with Royal Naval Reserve Sea Transport in the Middle East, Australia and Hong Kong. After the cessation of hostilities he sailed as chief officer on British ocean weather ships. In February 1949 he was appointed lecturer at the Plymouth School of Navigation, a post which he continued to hold until the time of his death.

E.L.H.

Corrigenda to October 1955 issue

R. S. Scorer—

(i) p. 555, Eq. (5.3)—

for $\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)$ *read* $\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)$

(ii) p. 557, § 6—

for $-\frac{\varpi}{\tau}$ *read* $+\frac{\varpi}{\tau}$ throughout the second paragraph of this section.

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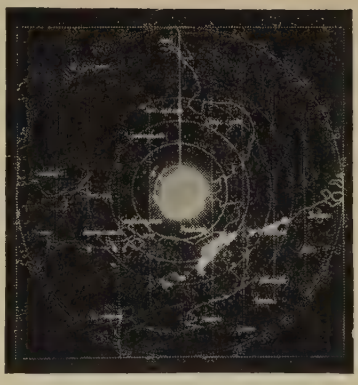

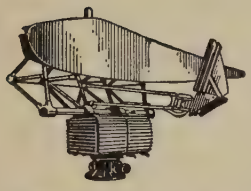

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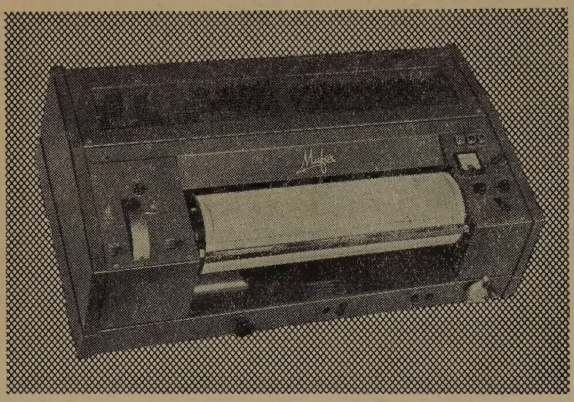

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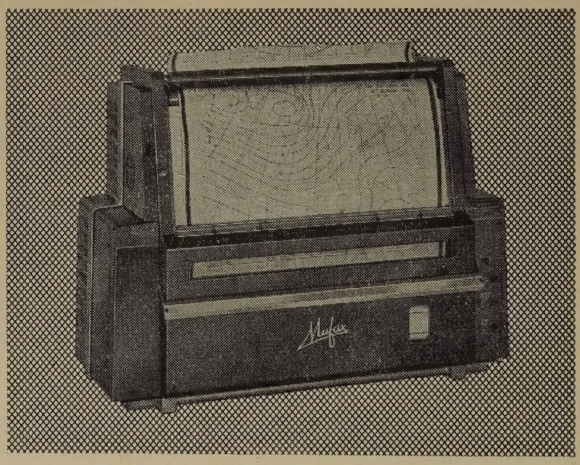

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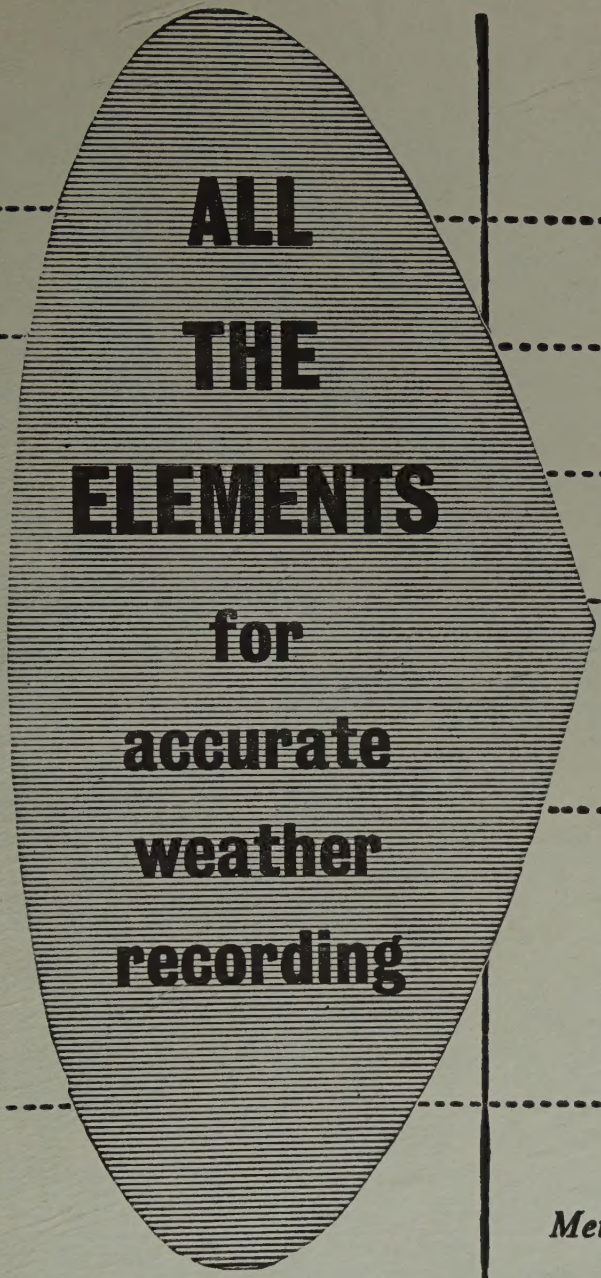

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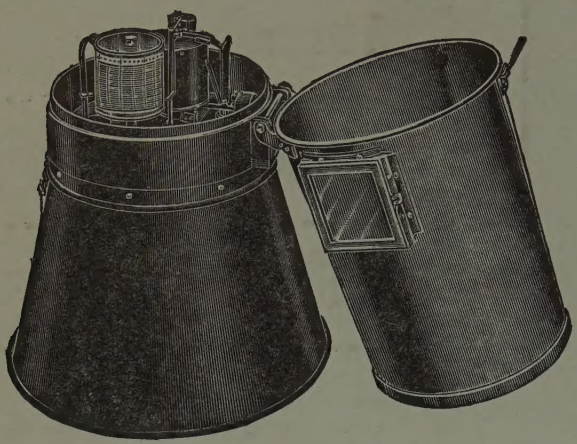

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